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THE EXTERNAL FORCES ON AN AIRSHIP STRUCTURE* WITH SPECIAL REFERENCE TO THE REQUIREMENTS OF RIGID AIRSHIP DESIGN

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§1. THE COMPONENTS OF A SYSTEM OF EXTERNAL FORCES

A system of external forces on the structure of a rigid airship can be analysed into two or more of the component systems due to

1 W. The Weights of the members of the structure and the weights of the masses carried by it;
2. L. The Lift and pressure of the gas;
3. A. The Aerodynamic forces consequent on the relative motion of the airship and the air;
4. T. The Thrusts of the airscrews;
5. R. The Reactions due to either the mooring arrangements, the hauling-in guys, the supports used in the shed, or handling on the ground.

As these component systems, though restricted by their interdependence, are individually variable, a very large number of complete systems is represented by the possible combinations and variations that can occur. It becomes necessary, therefore, for the purposes of design, to attempt to limit the number of combinations to be considered, and to restrict the variations of the components, but in such a way that the structure designed to carry the resulting systems with adequate factors of safety, shall be able to carry any other possible system with an appropriate factor of safety.

§1.1. *The Possible Combinations of the Components*

The possible combinations of the five components with the conditions for the ship to which they correspond, are

(a) Ship in the Shed
I W, R;
II W, L;
III W, L, R;
IV W, L, T, R;

* Submitted March, 1928.

(b) Ship in Free Flight I W, L;
II W, L, T;
III W, L_2, A, T;
IV W, L, A;

(c) Ship at Moorings I W, L;
II W, L, A, R;
III W, L, T, R;
IV W, L, A, T, R;

(d) Ship being moored I W, L, A, R;
II W, L, A, T, R;

(e) Ship being handled on the ground I W, L, R;
II W, L, A, R;

The number of these combinations cannot be conveniently reduced until the restrictions on the variations of the components have been defined.

§1.2. *The Variations of the Components*

§1.21. *Component W*

The weights supported fall naturally, as has been indicated, into two categories. The first, described as the Fixed Weight, consists of the essential structures and fittings, which cannot in ordinary circumstances be moved, and the second, known as the Disposable Weight, consists of all remaining items, and includes the load which the ship is designed to transport. The Disposable Weight can vary both in amount and distribution, though the portion which may be termed the Transport Load, if it consists of passengers, cargo, or mail, can only vary in distribution. If the Transport Load consists of aeroplanes, however, as it may in future airships, it will be possible for it to vary in amount during a flight, but this possibility will not be considered implicitly in this essay, as the means of taking it into account will be apparent, without additional explanation, from the procedures to be described.

When all the disposable weight which can be carried is in the ship, it is said to be in the Heavy Condition. This may be characterised as the condition of the ship at the beginning of the longest flight it can make when carrying its full Transport Load. The most extreme condition of an opposite character can obtain at the end of the flight, when all the disposable weight other than the transport load and the minimum safe reserve of ballast, has been used up or jettisoned. This condition may be termed the *Light Condition with Transport Load* on board.

The ship will not always be required to fly with the maximum transport load, so that at the opposite end of the scale of heavy conditions to the one described above, is one representing the conditions at the beginning of a flight of the same length with no transport load. This may be termed the *Heavy Condition with no Transport Load* on board. At the end of a flight under these initial conditions, the disposable weight left in the ship will be a minimum, and the ship is described as being in the *Light Condition.*

There doubtless exist, intermediate between these four, weight conditions equally severe in their effects on the ship's structure, but there is generally no reason to suppose that they could produce actions anywhere on it appreciably *more* severe than those occasioned there by at least one of the four clearly defined limiting conditions that have been briefly described. The design is safeguarded, moreover, by adequate factors of safety.

The primary restriction placed on the component system due to weight, therefore, is that its allowable variations shall be four only, defined as

Wa. The Heavy Condition.
Wb. The Heavy Condition without Transport Load.
Wc. The Light Condition with Transport Load.
Wd. The Light Condition.

In the combinations given in §1.1, therefore, W can generally be replaced by Wa, Wb, Wc, Wd, in turn. The weight distributions corresponding to the four conditions should all balance about a point in the ship which, when the ship's axis is horizontal, is vertically below the point marking the centre of lift when the gasbags are full, but modifications to these distributions are permissible in cases where the equilibrium of the complete loading system requires it.

§1.22. *Component L*

In normal circumstances the ideal arrangement is for the lift and weight to have such magnitudes and distributions that they together provide a system in static balance. It is reasonable, therefore, to adopt as the bases of possible variations in component system L, four conditions, which, when taken with Wa, Wb, Wc, and Wd, provide four systems in equilibrium. The primary variants of the lift component system can therefore be

La. All gasbags full, when the lift balances the weight for Wa.
Lb. All gasbags proportionately deflated until the lift balances the weight for Wb.
Lc. All gasbags proportionately deflated until the lift balances the weight for Wc.
Ld. All gasbags proportionately deflated until the lift balances the weight for Wd.

The lift of the gas may become unequal to the weight for various reasons, and its distribution may vary. The consequent possible inequalities and variations in the static force system are accounted for by the consideration of the effects of superheating and of certain conditions of gasbag deflation to be specified. The first of these conditions is that due to the complete deflation of any one gasbag. This produces for the portion of the structure near the deflated gasbag a distribution of static forces more serious than can be derived in any other way, and is, perhaps, more logically regarded as catering with some reserve of strength for the worst static force distributions that are likely to occur, rather than as catering for the remote possibility of accidental deflation.[1] Unless balance is achieved statically, the differences between total weight and static lift arising from the consideration of each gasbag completely deflated in turn for each of the variants La, Lb, Lc, Ld, provide a set which it is unreasonable far to exceed in building up cases for design; and the corresponding set representing the maximum design conditions for excess of lift over weight is equally reasonably provided by the maximum amount of superheating of the gas which can occur.

If the more literal view of complete gasbag deflation is taken, then what may prove for Lb, Lc and Ld a more serious distribution of static forces than that implied above, occurs if the deflation happens simultaneously with superheating, though the excess of weight over lift is less than when the gas and air temperatures are the same. This change in distribution cannot arise in the variant La, however, as the inflated gasbags are already full and gas is released from the automatic valves when expansion occurs, leaving the total lift the same. If, however, when all the gasbags are full, superheating has occurred and the gas has then cooled to the air temperature, the effect is one of the

[1] It has occurred once in the history of British airships.

proportional partial deflation along the ship. This state of affairs is realised to some extent if the ship enters a region of warm air, and, as it is also representative of the general heaviness to which an airship is liable from various causes, the case of proportional partial deflation should be examined for all four variants of the lift component system, though it is unlikely to prove serious. The simultaneous occurrence of gasbag deflation with this uniform deflation produces a greater difference between weight and lift than any other system, and in a list of variations of the L component which contains superheating with deflated gasbags, this variation should be included.

The maximum amount of superheating will occur when the gas volume is originally such that the expanded volume just fills the gasbags. If, before the superheating of the gas, the ship is in trim, a condition which may be considered to represent La with superheating is obtained, and may be considered with gasbags deflated in turn.

It would appear then that the possible inequalities between weight and lift, and the possible distributions of lift, are comprehended in a scheme of component systems L which, in addition to the systems La, Lb, Lc, Ld, includes the variations in Table 1.

TABLE 1.

VARIATIONS IN THE COMPONENT SYSTEMS La, Lb, Lc, Ld, TO BE CONSIDERED IN DESIGN.

1	La	with superheating
2	Lb	
3	Lc	
4	Ld	
5	La	with gasbags proportionately deflated
6	Lb	
7	Lc	
8	Ld	
9	La	with gasbags deflated in turn
10	Lb	
11	Lc	
12	Ld	
13	1	with gasbags deflated in turn
14	2	
15	3	
16	4	
17	5	with gasbags deflated in turn
18	6	
19	7	
20	8	

§1.23. *Component A*

The distributions of aerodynamic forces which it is possible for the ship to experience fall into three classes:—

1. Those due to steady relative motion of the ship and the air;
2. Those due to a relative motion of the ship and the air which changes in a steady manner;
3. Those due to an unsteady motion of the ship and the air.

These forces are conveniently resolved into directions parallel to three mutually perpendicular axes of reference fixed in the ship. These are shown in Fig. 1. Forces parallel to the axis OX are termed *Longitudinal Forces,* those

parallel to OY are termed *Lateral Forces*, and those parallel to OZ are termed *Normal Forces*.

When the axis of the ship and the direction of the relative wind are inclined to one another and both lie in the same vertical plane, the angle between them is the *angle of pitch* of the ship. The inclination of the ship to the horizon is measured by the algebraic sum of the angle of pitch and the angle between the horizontal and the direction of the relative wind or flight path of the ship. If the ship's axis is in the horizontal plane and at a point on it the relative wind is inclined to the direction of the axis there, the angle of inclination is the *angle of yaw* at the point in question.

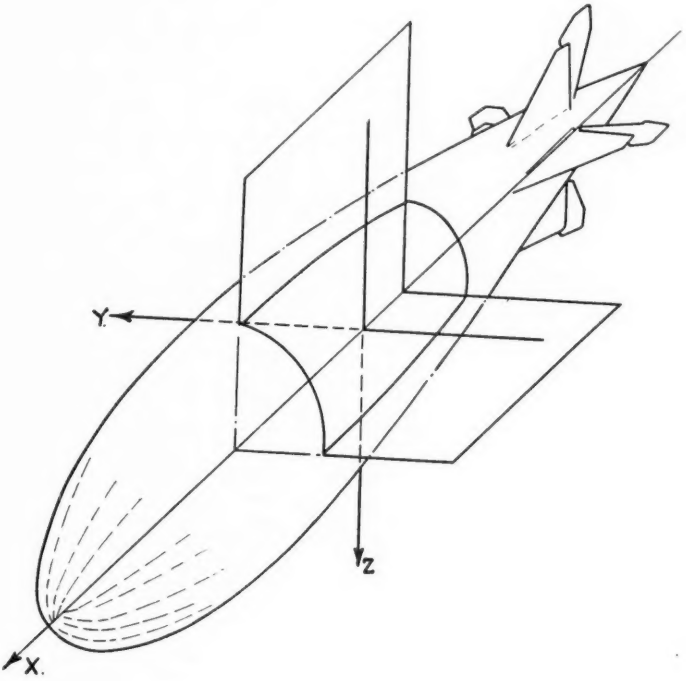

FIG. 1.

§1.231. *In Free Flight*

When the relative motion is steady, the angles of pitch and of inclination to the horizontal are constant, and have each the same value at all points in the ship. The forces produced can be ascertained fairly accurately, and can be resolved into longitudinal and normal components. For the angles of pitch which the ship is likely to have in flight, these components both increase with the pitch angle, so far as the hull is concerned, while the air speed is constant. In actual flight, however, a change of pitch has, among its consequences, changes in thrust and probably elevator angle, as well as a change in air speed, so that it is not possible by inspection of experimental data to select an angle of pitch which is likely to produce the most stringent conditions for pitched flight.

It becomes necessary, therefore, to consider the forces for various degrees of pitch at the maximum possible speeds. The angles suggested are 0°, 5°, 10°, 15°, but numerical work on these may indicate the advisability of considering other angles as well.

When the ship is performing a steady turn its path is circular, while the angle of yaw varies continuously from nose to tail. This case also can be satisfactorily dealt with, and produces forces which resolve naturally into longitudinal and lateral components. As in pitch, it does not follow that the forces are most severe when the yaws are greatest, so that it is advisable to consider circling flight both at maximum rudder angle and at some smaller rudder angle, at the probable maximum speeds in each case.

It is possible for the ship to have simultaneously both angles of pitch and yaw. It is suggested, however, that this case, which is extremely difficult to analyse satisfactorily, is unlikely to produce conditions which are not covered by the worst pitch and circling flight cases, for the speed of the ship, lessened by both pitch and yaw, must necessarily be extremely low. The question is discussed further in Appendix I.

Accelerated motions in free flight can arise in several ways. The most important are :—

1. The acceleration[2] consequent on switching off the engines;
2. The acceleration consequent on starting the engines;
3. The angular and linear accelerations due to attaining or returning from an angle of pitch;
4. The angular and linear accelerations due to going into or getting out of a turn;
5. The angular and linear accelerations consequent on coming into contact with gusts.

In the first two cases it is difficult to imagine lateral or normal forces greater than those arrived at in the steady motion cases. The inertia forces are mainly longitudinal, so these cases are adequately dealt with by considering longitudinal forces for the ship in horizontal motion only.

Cases 3 and 4 can, with certain assumptions, be treated as step-to-step calculations, but are more conveniently dealt with by assuming the elevators and rudders respectively moved rapidly from amidships to the hard-over position. It is concluded in §16 that the conditions, while getting into a steady turn, are intermediate between those for rapid rudder movement and steady turning, and a corresponding conclusion may be reached in the case of pitch. Any attempt to consider the case of rapid movement of the rudders from one hard-over position to the other while in a steady turn will not result in more severe actions. It is a less accurate investigation, and will not be considered, though the treatment would be on the lines of §16.

With regard to case 5 above, the information on gusts is at present meagre, and calculations cannot be made with sufficient accuracy to warrant their application to actual design. Calculations on severe assumptions, some of them along the lines suggested by Mr. Burgess,[3] indicate, however, that gust conditions have a reasonable factor of safety in a design produced without implicit reference to them, and experimental information recently obtained supports this view.

§1.232. *Ship at the Mast*

When the ship is at the mast, and a steady wind is blowing, the worst aerodynamic conditions that can occur depend on the maximum degrees of pitch and yaw which the ship can attain and the maximum wind speed at which it is considered safe to remain moored. The maximum degree of yaw depends on

[2] Negative.

[3] U.S.A. National Advisory Committee for Aeronautics, Report No. 204.

the maximum possible rudder angle. The primary purpose of the rudder is the turning of the ship, and there is, in consequence, no particular reason why full rudder should occur at the mast, and thereby produce a most severe condition of loading. It has been suggested, therefore, that arrangements be made to limit mechanically the possible rudder angle at the mast. There is only one reason, apparently, why the rudders should not be locked amidships, and that is that most ships are not stable at the mast, and need a small rudder angle to prevent hunting. It seems, therefore, that the allowable rudder angle must be either the angle of maximum travel, an angle sufficient for stability, or some arbitrary angle intermediate between these. The choice will affect the factor of safety to be associated with the case.

The maximum degree of pitch depends on the elevator angle, the trim of the ship, and the possible temperature gradients, together with any automatic means which are designed to limit the pitch. A reasonable limiting figure is 5°, as before the ship had attained such an inclination measures would have been taken to arrest it.

The worst aerodynamic condition exists when the ship has at the same time its maximum pitch and maximum yaw at the maximum wind speed decided on. This speed is given as 60 m.p.h. by the Airworthiness of Airships Panel[4], though a designer may have reasons for choosing a higher.

Gusty wind conditions at the mast can cause severe lateral loading on the ship, which is especially important in the region at the nose, and if gusts are treated as rapid, but steadily varying changes in the wind, it is possible to calculate their effects on lateral forces in certain simple cases. The necessary theory for the cases of

1. " A wind of constant linear velocity changing in direction with constant angular velocity."
2. " A wind constant in direction increasing from zero velocity with constant acceleration, some initial angle of yaw being assumed;"

is given[5] in Appendix III. to " A Review of the Present Position with Regard to Airship Research and Experiment," by Lt.-Col. V. C. Richmond.

These conditions represent the effect of gusts which are sufficiently large to operate simultaneously on the whole ship.

§1.233. *Ship being Moored*

With the latest methods for bringing the ship on to the masthead, described by Major Scott in his paper on " The Development of Airship Mooring,"[6] the possible aerodynamic conditions are sufficiently like those when the ship is actually attached to the masthead to need no independent calculations. The direction of the reaction at the nose can be different, however, and the effect of this needs examination.

§1.234. *Ship being Handled on the Ground*

It must be assumed that the ship will *not* be handled on the ground in a wind greater than 20 m.p.h. The effect of this wind blowing broadside on to the ship may be considered.

§1.235. *Variation of Component A*

Summarising, the variations of the component system A to be considered are those given in Table 2.

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[4] R. & M. 970, p. 10.
[5] Journal of the Royal Aeronautical Society, Vol. XXX., p. 577.
[6] Journal of the Royal Aeronautical Society, Vol. XXX., p. 465.

TABLE 2.

VARIATIONS IN THE COMPONENT SYSTEM A TO BE CONSIDERED IN DESIGN.

Relative Motion Steady.

1. Ship in free flight, travelling at the maximum speed possible at pitch angles 0°. . .15°.
2. Ship in free flight, circling at the maximum speed possible with maximum rudder angle and at some smaller rudder angle.
3. Ship at the mast with maximum pitch and wind speed 60 m.p.h.
4. Ship at mast with maximum yaw and wind speed 60 m.p.h.
5. Ship at mast with maximum pitch and maximum yaw, and windspeed 60 m.p.h.
6. Ship on the ground, with broadside wind of 20 m.p.h.

Relative Motion Changing Steadily.

7. Ship in free flight, decelerating at 0° pitch.
8. Ship in free flight, accelerating at 0° pitch.
9. Ship in free flight, in act of pitching.
10. Ship in free flight, getting into or out of a turn.
11. (Ship in free flight, meeting gusts).
12. Ship at mast, wind veering.
13. Ship at mast, wind accelerating linearly.

§1.24. *Component T*

The thrust is definitely settled by the aerodynamic requirements. In Table 2 the requirements of maximum speed in the cases 1, 2, 8, 9 and 10 decides the thrust. Case 7 requires the maximum thrust at no speed of advance, and the remaining cases have no thrust component.

§1.25. *Component R*

The nose reaction due to mooring or hauling-in is the most important reaction. It is determined by the other forces operating. The reactions in handling the ship on the ground are not likely to prove important except for local fittings, and the same remark applies to shed reactions.

§2. FACTORS OF SAFETY

Having reviewed the variations of the component systems, and indicated which of them on their own merits require examination, it becomes necessary to expand the combinations given in §1.1. into the cases which should be considered for the purpose of design, and to allot to these cases suitable factors of safety.

If the distributions of loading of the structure considered for design were all equally likely to occur in practice, then the factor of safety in each case would be the same. Actually, however, some distributions are more likely than others, and it is logical to allot to the stressing actions representing a frequent occurrence, a higher factor of safety than that allotted to the actions resulting from a less frequent occurrence. This view is strengthened by the fact that any chosen distribution of forces is no more than an average of distributions actually occurring in the circumstances from which it is derived. That is to say, if in a given airship, a certain gasbag is supposed deflated and then static balance is achieved, while maximum speed is maintained, the distribution of weight, lift, and aerodynamic force may be made precisely the same as the designer has assumed in his calculations for the case; but in practice, due to the movement of passengers and crew, the consumption of fuel, the exigencies of manœuvring, the unsteadiness of the atmospheric conditions and other possibilities, the forces on the structure will vary in distribution, and in consequence, in almost any structural member, the stress will fluctuate about a mean which is probably close to the

figure calculated for the case. Although these fluctuations are probably disordered and intermittent, and unlike what is generally meant by an alternating stress, the idea of fatigue is introduced, and the case for allowing " probability " to control the factor of safety is strengthened. Another consideration is that if in any case there is reason to suppose the fluctuations are greater than the average, the factor should be correspondingly raised. This view can be argued in the case of gusts, where the factor should be higher than that arrived at on the grounds of infrequency.

These considerations are qualitative, and no quantitative rule can be suggested for obtaining the factor. Its selection must be to a large extent arbitrary, though guided by the foregoing considerations, by experience, and by the conclusions of the Airworthiness of Airships Panel[7] on the matter.

§3. The Systems of External Loading to be Considered in Design

§3.1. *Ship in Shed*

§3.11. W.R. Combination a1 in §1.1, in which the ship is without gas in the shed, is not likely to provide severe conditions of shear, bending, or transverse distribution, as there will be supports arranged at every frame and under important concentrations of mass, such as power cars. The case need not be dealt with therefore unless special supporting arrangements or other particular reasons make it necessary. The method of developing the primary actions for the case in such circumstances will be apparent from the detailed treatment of ensuing cases.

§ 3.12. W.L. During the gassing of the ship, it will continue to be held in such a way that no important primary actions can arise. It is possible, however, for a passenger flight to start from the shed, so the heavy conditions represented by (Wa, La) (Wb, Lb) are possible; similarly, the possibility of flights ending in the shed makes conditions represented by (Wc, Lc.), (Wd, Ld) possible also. Except perhaps in the heavy condition, the gasbag deflation conditions can also be considered to occur, but it will be seen that all the (W, L) shed conditions can be covered by free flight conditions.

§3.13. W.L.R. If the weight is unequal to the lift in the shed, it may be assumed that the local differences at transverse frames will be made up by reactions applied there, so that the shears across bays will remain approximately zero, and the bending moments correspondingly small.

§ 3.14. W.L.R.T. In the event of a test of the power plant in the shed, it may be assumed that the vertical forces are approximately balanced at every frame. Only if longitudinal reactions are not applied *at* the engine bearers can the whole structure remain unstressed, and the manner in which the structure is stressed depends on where the reaction to prevent movement of the ship is applied. This is not a case[8] which affects the whole ship, but it needs examination for the structural members which carry the power plant, with, it is suggested, a minimum factor of safety of 3.

§3.2. *Ship in Free Flight*

§3.21. W.L. The ship can be stationary in the air, and the condition is approximated to when it rises from, or lands slowly to the ground. All the conditions represented by Wa, b, c, d, can occur, and the gasbag deflated conditions, 5-8, 9-12, 17-20 of Table 1 can be supposed. In these latter cases, as there is no forward relative motion between the ship and the air, equilibrium must

[7] R. & M. 970, p. 9.

[8] It is probably, in any case, covered by Cases 89-104. §13.

exist between the weight and gas lift systems, so the weight distributions Wa, b, c, d must be modified by dropping and moving disposable weights. The conditions (Wa, La) . . . (Wd,Ld) may reasonably be described as normal conditions to be frequently expected, and as such should have applied to them a factor of safety of 4.[9]

A factor of 3 on the cases with single gasbag deflations is in proportion; cases with proportional deflation, however, represent circumstances of more frequent occurrence, and require therefore a factor of safety intermediate between 4 and 3: it may be fixed at 3.5.

§3.22. W.L.T. When in one of the foregoing circumstances the engines are started, the condition corresponds with the accelerated motion (2) in §1.231, the initial velocity being zero.

This is the only example of §1.231 (2) that need be considered, as to deal with other initial velocities is found to be an unnecessary refinement. The vertical forces, in normal and gasbag deflated conditions, remain unaltered, but there is superimposed in each case a set of longitudinal forces due to the thrust and the ship's inertia. The thrust will cause a moment tending to pitch the ship, and there will be an additional pitching moment if the centre of mass is offset from the ship's axis. No weight adjustment to balance these moments need be contemplated however, as the pitching tendencies will be resisted by further inertia forces. There is no need to calculate these longitudinal forces for all the distributions of static forces represented by the various gasbag deflation causes. Experience has shown that longitudinal forces are much less important in design than the primary shearing and bending actions; the loads they produce in the structure are not of the same order of magnitude. It is, in consequence, sufficiently accurate in the present instance to calculate the longitudinal force systems using the inertia forces corresponding to the four mass distributions of the cases (Wa, La). . . . . (Wd, Ld) only. All (Wa) cases, with no deflation, uniform deflation or local deflation, have then superimposed on them the same longitudinal forces due to starting the engines, and Wb, Wc, and Wd cases are treated similarly. The conditions in these cases are no more abnormal than those described in §3.21, and the factors of safety given are unaltered when these longitudinal actions are added.

§3.23. W.L.T.A. Under the thrust, the ship finally attains a state of steady motion. The first aerodynamic conditions to be examined are those at 0° pitch and maximum speed. The weight and lift forces first to be considered are (Wa, La) . . . (Wd, Ld), which need some weight adjustment to neutralise the moment due to the thrust, and the drag of the cars. Longitudinally, the drag of the ship is balanced by the thrust of the propellers. These represent normal working conditions, and as such, need a factor of safety of 4. When proportional deflation, or superheat occurs, or when a gasbag is considered deflated, at 0° pitch it is not generally possible to supply the deficiencies in force and moment by means of the elevators alone, and the static conditions must be further modified by dropping weight or valving gas as the case may be, and moving weights. For 0° pitch, it is safe to suppose the elevators always amidships, and to achieve equilibrium by static means only, the effect of elevator angle being adequately covered by the next W.L.T.A. cases to be considered. In the present cases the factors of safety for local deflations and proportional deflations should remain 3 and 3.5 respectively. No especial significance attaches to superheating in this instance as gas is valved and the lift remains constant.

At angles of pitch such as 5°, 10° or 15° it becomes possible to balance deflation by dynamic lift; there is then no need to drop any weight, though it may need some redistribution to give balance of moments. Negative pitch may likewise be used to balance the " false lift " due to superheating. It is clearly

[9] R. & M. 970.

possible to provide balance in a given case by a combination of static and aerodynamic means, but as it is proposed to consider the purely static and purely aerodynamic means in detail, the additional complication of any intermediate examples must not be considered. It does not appear possible that any hybrid cases could serve any useful purpose; their construction would be very arbitrary and there is no evidence that any case intermediate between the two extremes defined, would supply more severe actions when applied with an appropriate intermediate factor of safety, than they supply. The use of aerodynamic means of maintaining equilibrium alone, represents rarer circumstances than the use of static means, and the factor of safety on local and proportionate deflation cases may now be 2.5 and 3 respectively.

It is not considered necessary to analyse the motions of the ship while it is attaining the necessary angle of pitch,[10] but the case (3) of § 1.231 may be interpreted, as there indicated, by considering the initial stages of the motion. Four representative cases corresponding with the conditions Wa. . . d should be sufficient, as it is in the tail region that the actions are likely to be important, and the variation in the forces occasioned by performing the operation with different gasbags deflated will be sufficiently accurately represented by a superimposition of force systems as indicated later.[10] These cases represent a definitely unlikely occurrence, and the calculations proposed in § 15 presume an instantaneous elevator movement. It is, in consequence, fair to assume a low factor of safety, say 1.5, when the superimposed system is associated with a case of complete deflation of a gasbag.

Just as when the ship was considered to be getting into pitched flight conditions the investigation of the initial stages of a motion incurred by a sudden change in elevator angle was deemed sufficient, so when it is getting into a turn, the conditions may be safely considered by the examination of the initial stages of the motion caused by putting the rudders to hard-over from amidships. It is considered that the superimposition of the resulting force system on those for the o° pitch, maximum speed cases, with factors of 2, 1.75, and 1.5 respectively for full, proportionately deflated, and deflated gasbags, is sufficient.

When the ship has attained a steady circling motion, the various static conditions for o° pitch must be considered in conjunction with the aerodynamic and centrifugal forces consequent on the two degrees of rudder angle chosen. The cases which involve the static conditions which, in rectilinear flight at o° pitch, had respectively factors of 4, 3.5 and 3, may now have factors of 3, 2.75 and 2.5.

§3.24. *W.L.A.* If when the ship is proceeding steadily, the engines are switched off, inertia forces are introduced which affect the longitudinal forces on the ship. The circumstance needs examination in the cases (Wa La) . . . (Wd Ld) only, as in other cases longitudinal forces will be a little different. The longitudinal forces may be superimposed on all the systems for o° pitch but not on those for other pitches, as the engines are not likely to be switched off when the speed is needed to provide aerodynamic lift. The factors of safety will be those for o° pitch.

§3.3. *Ship at the Mast*

§3.31. W.L. If the ship is in static trim at the mast, and with no wind blowing, the forces are the same as those of §1.321.

§3.32. W.L.A.R. If the ship is in static trim at the mast in a wind, the previous case is modified by the addition of longitudinal aerodynamic forces, a reaction at the masthead, and some adjustment of the elevators to balance

[10] §15.

any asymmetry of the drag forces due to cars, etc. The appropriate factor of safety is 4.

If any large difference between weight and buoyancy arises, it must be always considered to be balanced statically, as suitable aerodynamic forces are not available if the wind speed is low, and the limitations on the pitch restrain their use when it is not. So pitch systems at the mast must consist of systems due to static forces in balance, and of aerodynamic forces due to a 60 m.p.h. wind in balance as regards moment, but requiring a masthead reaction for vertical balance. These cases represent conditions which can easily occur at the mast, and so the suitable factors of safety appear to be 3.0 when the gasbags are as in La . . d, 2.75 for cases with uniform deflation, and 2.5 on the cases with gasbags deflated in turn.

When the ship is expected to ride for any length of time at the mast special means are taken to ensure that it shall always be in static trim. This is especially necessary in tropical climates, where temperature gradients can be extremely severe. An arrangement that has been successfully adopted, and which will be developed, is to attach heavy weights to the ship in such a way that, being trimmed light for the purpose, it partially supports them while the ground supplies the remaining forces to balance their weight. In practice, pairs of artillery wheels have been used, attached to the ship by lines which have a strong spring in their length. Their ability to deal with gain or loss of lift occasioned by a temperature gradient has been dealt with in Appendix III to Major Scott's paper on "The Development of Airship Mooring."[11]

If the temperature gradient causes a false lift, the ship tries to lift the wheels, its tendency to rise being checked by the tension in the lines which, by virtue of the springs, increases until a state of equilibrium is attained. If the gradient is of opposite sign, and the tail tends to drop, the tension is relieved until a state of balance is reached. With wheels of the correct weights and properly designed springs, the amplitude in pitch can be very restricted. For this reason, and because no appreciable wind is likely with a severe gradient of temperature, the aerodynamic loads are negligible.

The case of the ship trying to lift is the more important, as then the local reactions due to the wheels are greatest. The effect is more severe than that calculated on the superheating basis, so the latter is covered. Consideration of the four conditions represented by (Wa, La) . . . (Wd, Ld) is sufficient in this instance, as deflation of gasbags reduces the reactions and hence the severe shears produced by the attachments. The case seems to merit a factor of safety of 3.

The necessary cases of pure yaw to be investigated consist simply in the combination of the aerodynamic forces at a wind speed of 60 m.p.h. for the maximum yaw with the static conditions of o° pitch. The severity of the cases is determined by the maximum rudder angle allowed, but in any case, remembering that in the absence of the yaw, the factors of safety would be (§3.21) 4 for full and 3.5 and 3 for proportional and local deflation cases. The corresponding factors in the yaw cases for the mast should be 3, 2.75 and 2.5.

The condition of simultaneous pitch and yaw at the mast is not correctly obtained by superimposing the forces due to pitch and yaw taken independently, but when the true system of aerodynamic forces is obtained it can be superimposed on the balanced static systems. The coincidence of the maximum pitch and yaw conditions is not a very likely condition, and if it occurs, it would be quickly corrected, so factors of safety for the three classes of cases may reasonably be fixed at 2.5, 2.25 and 2.

§3.4. From the foregoing remarks, it appears necessary to consider in design the systems of external forces given in Table 3.

[11] *loc. cit.* §1.233.

TABLE 3.

SYSTEMS OF EXTERNAL FORCES TO BE CONSIDERED IN DESIGN.

No. of Case.	Description of Case.	References.	Factors of Safety. Gasbags as in La . . . d.	Gasbags proportionately defl.	Gasbags comp. defl. in turn.
	Ship horizontal and stationary relative to the air in				
1	Heavy condition with all gasbags full	(Wa, La)	4		
2	Heavy condition without transport load with all gasbags inflated ...	(Wb, Lb)	4		
3	Light condition with transport load with all gasbags inflated proportionately with sufficient gas for equilibrium	(Wc, Lc)	4		
4	Light condition with all gasbags inflated proportionately with sufficient gas for equilibrium ...	(Wd, Ld)	4		
5	Case 1, modified by uniform deflation	*cf.* Table 1, 5	3·5		
6	Case 2, modified by uniform deflation	,, ,, ,, 6	3·5		
7	Case 3, modified by uniform deflation	,, ,, ,, 7	3·5		
8	Case 4, modified by uniform deflation	,, ,, ,, 8	3·5		
9	Case 1, with each gasbag considered deflated in turn	,, ,, ,, 9	3		
10	Case 2, ditto	,, ,, ,, 10	3		
11	Case 3, ditto	,, ,, ,, 11	3		
12	Case 4, ditto	,, ,, ,, 12	3		
13	Case 5, ditto	,, ,, ,, 17	3		
14	Case 6, ditto	,, ,, ,, 18	3		
15	Case 7, ditto	,, ,, ,, 19	3		
16	Case 8, ditto ... ... the equilibrium in each case being maintained statically.	,, ,, ,, 20	3		
17-32	As in cases 1-16, but with the ship travelling at maximum possible speed at 0° pitch	,, ,, ,,	4	3·5	3
33-44	As in cases 5-16, but with the ship travelling at maximum possible speed with various angles of pitch θ ($0° < \theta < 15°$), equilibrium being maintained aerodynamically ...	,, ,, ,,		3	2·5
45-52	As in cases 1-4, 9-12, when modified for superheating, but with the ship travelling at maximum possible speed with various angles of pitch θ ($0° > \theta > -10°$), equilibrium being maintained aerodynamically	,, ,, ,,	3		2·5
53-68	As in cases 1-16, but with ship turning steadily at maximum estimated speed with maximum rudder angle, and also at some smaller rudder angle	,, ,, ,,	3	2·75	2·5

No. of Case.	Description of Case.	References.	Factors of Safety. Gasbags as in Lt. d.	Gasbags proportionately defl.	Gasbags comp. defl. in turn.
69-84	As in cases 1-16, but with the ship at the mast with the wind blowing 60 m.p.h. :—				
	(*a*) With no pitch or yaw ...	,, ,, ,,	4		
	(*b*) Pitched $+5^\circ$	—	3	2·75	2·5
	(*c*) Pitched -5°	—	3	2·75	2·5
	(*d*) With maximum allowable yaw	—	3	2·75	2·5
	(*e*) With maximum allowable yaw and $+5^\circ$ pitch ...	—	2·5	2·25	2
	(*f*) With maximum allowable yaw and -5° pitch ...	—	2·5	2·25	2
85-88	As in cases 1-4, when modified for superheating, but with the ship at the mast	—	3		
89-104	As in cases 1-16, but with forces due to starting the engines superimposed	—	4	3·5	3
105-120	As in cases 17-32, but with forces due to switching off the engines superimposed	—	4	3·5	3
121-136	As in cases 17-32, but with forces due to rapid change of elevator angle superimposed	—	2	1·75	1·5
137-152	As in cases 17-32, but with forces due to rapid change of rudder angle superimposed	—	2	1·75	1·5
153-168	As in cases 1-16, but with the ship at the mast in a veering wind ...	—	2	2	2
169-184	As in cases 1-16, but with the ship at the mast in an accelerating wind	—	2	2	2
	(Ship on the ground in broadside wind of 20 miles an hour)	§1.234			

§4. Primary and Secondary Stresses

The main hull structure which carries the loads is generally a framed structure, consisting of longitudinal members and transverse frames arranged as shown in Fig. 2, braced together with wires.

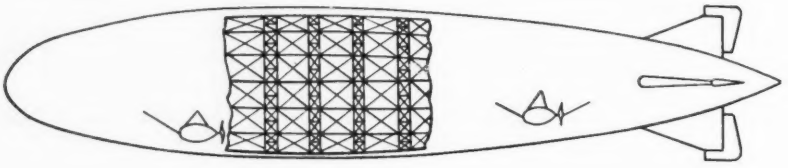

Fig. 2.

Any member of this structure may experience internal stresses of two kinds, which are designated primary and secondary stresses.[12] The former are obtained by considering the effect on the structure, supposed pin-jointed, of an external

[12] For definitions of primary and secondary stresses see R. & M. 970, p. 6.

load system whose component loads are made to act at the joints of the structure. The fact that any particular external load is not applied at a main joint, but at some intermediate point on a structural member, gives rise to secondary stresses in that member, which, together with other secondary stresses that arise, are algebraically added to the primary stress due to direct end load obtained for the member. This may be made clearer by considering the action of the weight of, say, a longitudinal girder between two adjacent frame faces. If it is of constant section, then, for the purpose of obtaining the primary internal stress, its weight is completely represented in the external loading system by point weights at its end joints equal to half its weight. These play their part in obtaining for the girder the primary end load which produces the primary stress in it. But the fact that the weight is actually an evenly distributed load on the girder has to be taken account of, and the girder must, therefore, be considered as a beam under such a load, with the result that bending and shear stresses are found, which add algebraically to the stress due to the primary end load. These bending and shear stresses are considered to be secondary stresses. The severity of a given case of loading is due first of all to the primary actions of shearing force, bending moment, and longitudinal force it produces; secondly, to the magnitude of the distributed loads such as girder weight, and the aerodynamic and fabric tension loads that are transmitted to the girders; and, thirdly, to the distribution round the sections of the ship normal to the axis of the forces that produce the primary actions. There is not necessarily any connection between the worst primary actions at a certain section of the ship and the worst transverse distribution there; it is possible for both shearing force and bending moment to be zero at a particular frame, and yet for the transverse distribution of lift and weight forces to have serious secondary effects.

In consequence, the question of transverse distribution is not investigated for every case of primary loading. The problem is approached rather by considering the worst types of transverse distribution that can occur at a particular section of the ship, and developing the conditions that could produce them; but the decision as to what *are* the worst types is so closely associated with the characteristics of a particular structure as to be outside the scope of this paper.

As the joints of the members of the hull structure occur in general at the faces of the transverse frames, it follows that for a primary loading system the shear is constant between frame faces. Thus from end to end of the ship a set of primary shears form, if plotted, a diagram composed of horizontal lines drawn between perpendiculars set up at points representing the positions of the frame faces as shown in Fig. 3. The corresponding bending moment diagram consequently consists of various sloping straight lines between these perpendiculars as shown in Fig. 4.

§5. Estimation of Static Loading

Before cases 1-16 can be analysed, it is necessary to estimate the magnitudes and distributions of the weights and the lift forces due to the gas. Briefly, the fixed weight consists of

1. The main hull and tail structure;
2. Subsidiary structure for carrying engines, for the control car, and for accommodation purposes;
3. Gasbag wiring;
4. The cover and the gasbags;
5. The engines, propellers and accessory gear;
6. Tanks and pipe lines for water ballast, fuel and oil;
7. Miscellaneous items such as flooring, wireless equipment, mooring and handling gear, controls, instruments, walkways, ladders, etc., etc.;

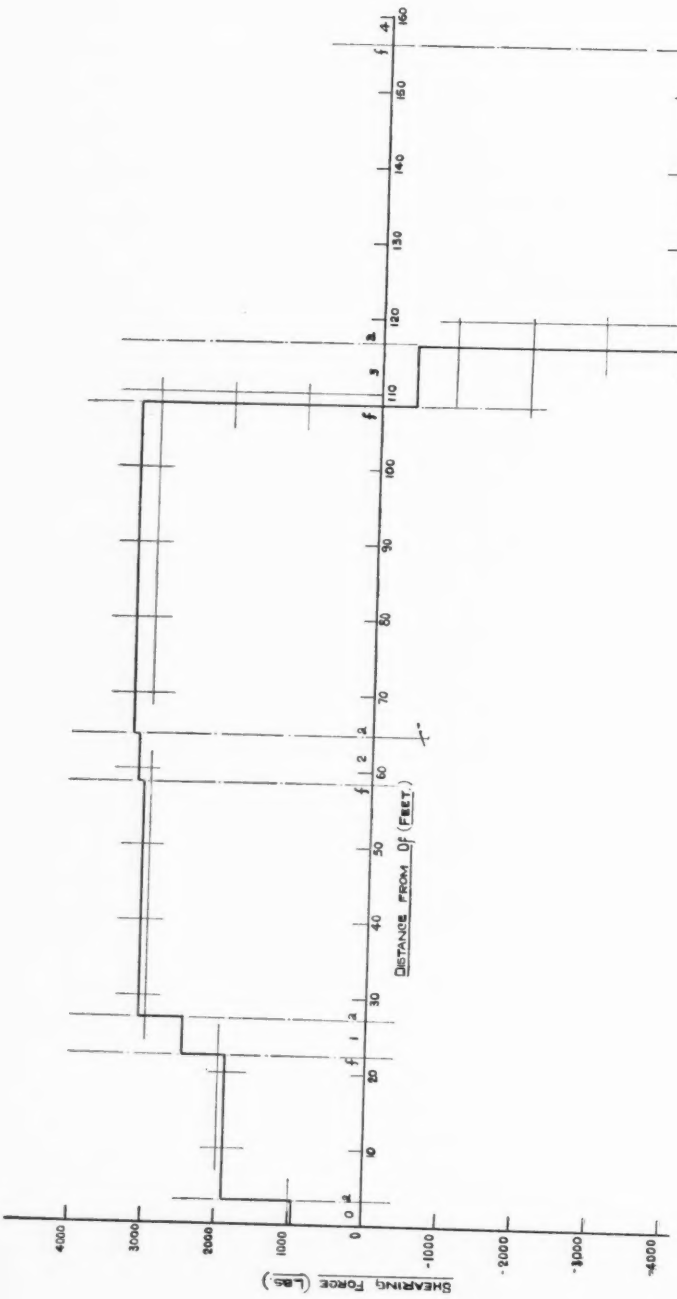
SHEARING FORCE (LBS)
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
DISTANCE FROM 0 (FEET)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160

FIG. 3.

and the disposable weight consists of

8. Water ballast, fuel and oil;
9. Crew;
10. The transport load—passengers, baggage, freight, mails, etc.;
11. Miscellaneous items such as bedding, parachutes, fire extinguishers and furniture.

The estimation of the weights of the items is, in the initial stages of design, a matter of experience coupled with a knowledge of the weights of corresponding items in previous airships, of the proposed power plant and range, of the service for which the airship is intended, and of the accommodation to be provided, while the distribution of the weights is carried out with an eye on the minimising of shearing, bending and distorting actions; no more can be said on the matter here.

The lift of the gas is usually taken as 68 lbs.[13] per 1,000 cu. ft., though pure hydrogen has a lift of 72 lbs. per 1,000 cu. ft.; the volumes of the various gasbags being known, their lifts can be found.

§6. Cases 1-16 (Cases of Static Loading—Ship Stationary)

§6.1. *Case 1*

In Case 1, all the weights the ship can carry are supposed on board in their appointed positions, and all the gasbags are just full. The condition for balance, which is that the C.G. of the weights must lie vertically beneath the C.B. of the lifting forces, must be fulfilled. It will have been the designer's object in addition to distribute the weights and lift forces in such a way that shearing forces are, consistent with the condition of equilibrium, as small as possible everywhere; that is, if the main weight and lift forces occur at the transverse frames, he will have made the difference between them at each frame as small as possible.

The first step in obtaining the primary actions is to consider the weights in each bay in turn, and, according to the positions of their centres of gravity, to represent them by two loads, at the centre lines of the frames at each end of the bay. When this has been done, there will be for each frame, a weight figure representing the loads occurring at the frame and its portion of the weights in the bays on either side of it.

TABLE 4.

1. Frame Face.	2. Bay or Frame.	3. Weight (lbs.)	4. Lift (lbs.)	5. Weight-Lift (lbs.)	6. Shear (lbs.)	7. Distance corres. with (2) (feet)	8. (6) × (7) (tons ft.)	9. B.M. (in tons)
of		1110	166	949				
	of-oa				949	3.28	1.4	
oa		1110	116	949				1.4
	oa-1f				1898	18.88	16.0	
1f		1613	1011	602				17.4
	1f-1a				2500	4.93	5.5	
1a		1613	1011	602				22.9
	1a-2f				3102	31.25	43.3	
2f		3543.5	3462	81.5				66.2
	2f-2a				3183.5	6.04	8.6	
2a		3543.5	3462	81.5				74.8
	2a-3f				3265	43.8	63.9	
3f		2517	6242.5	−3725.5				138.7
	3f-3a				−460.5	7.86	−1.6	
3a		2517	6242.5	−3725.5				137.1
	3a-4f				−4186	39.5	−73.9	
4f								63.2

[13] The use of this figure allows the ship sufficient lift when the purity of the gas has deteriorated.

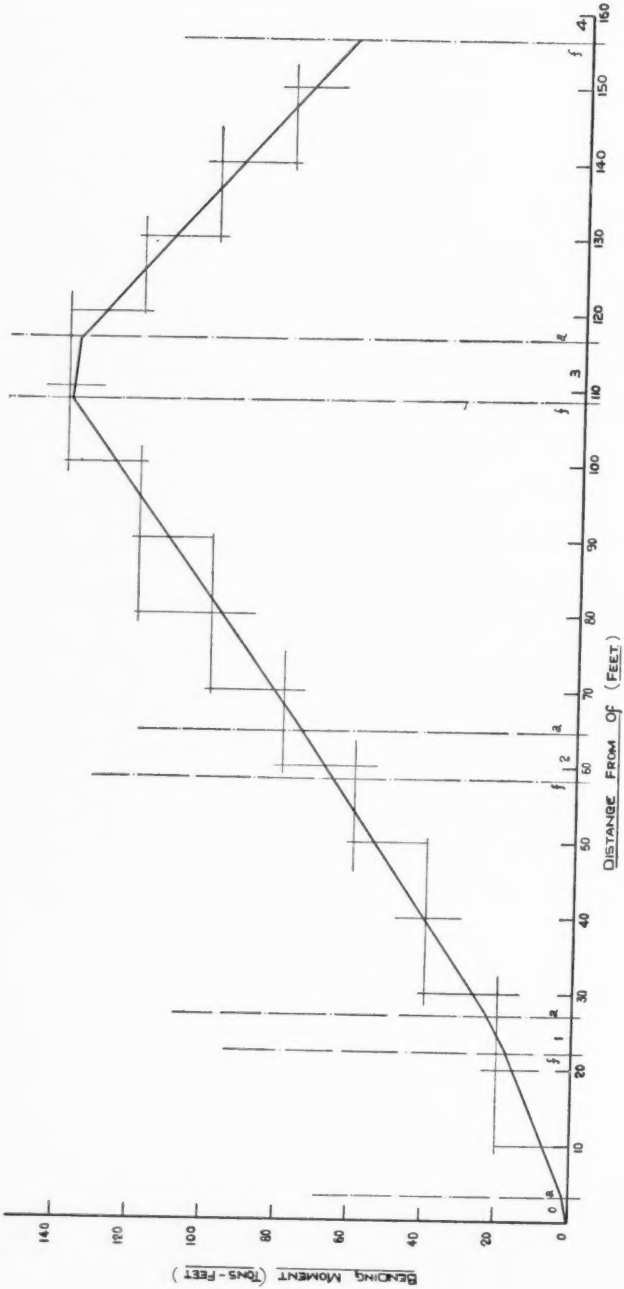
BENDING MOMENT (TONS-FEET)
0
20
40
60
80
100
120
140
DISTANCE FROM O (FEET)
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160

FIG. 4

The lift forces are then dealt with in a strictly similar manner. Two columns of weights and lifts may then be written down as shown in Table 4, where the frames are represented as having width, and consequently two faces on which the loads are supposed equally distributed. If there is any real tendency to asymmetrical loading, the loads in the bays and in the frames are replaced by loads at the centre lines of frames faces instead of by loads at the frame centre lines.

In column 5 the difference between weight and lift is written down. Then the *shearing force* across a given bay or frame, written in column 6, is given by the algebraic addition of the $(W\text{-}L)$ values for all frame faces forward of, or aft of, the given bay or frame. This column is represented diagrammatically in Fig. 3.

In column 7 the distances between adjacent frame faces are written. Then the products of columns 6 and 7 gives the *change* in bending moment between adjacent frame faces. The sum of these products occurring forward of, or aft of, a given frame face, gives the *bending moment* at that frame face. The bending moments are given in column 9 and represented diagrammatically in Fig. 4.

If the passengers and crew are allowed considerable latitude in movement, then othere xamples of the heavy condition, with these items concentrated in such ways as to produce the most severe local differences between weight and lift, should be written down, remembering that adjustments must always be made to keep the position of the centre of gravity the same.

The primary actions having been arrived at, the secondary external loading associated with them must be investigated. The longitudinal girders and the transverse girders of the frames will be stressed by their own weights, by local masses not attached at main joints, by loads due to the manner in which the gas lift is transmitted to the structure, and by loads due to the method of attaching the fabric, whose weight and tension are important items. In addition there will be secondary stresses due to the distortion of the structure under the applied loads, as has been mentioned in §4, but as stated there, these effects are treated separately. The loads on a member, due to its own weight and to fabric loads, can be expressed as a system of distributed loads. The gas also provides distributed loading on the longitudinal girders in the majority of airships, though it is possible to design gasbag wiring systems which produce only direct end loads in the members of the structure.[14]

It will generally be found that the secondary loading varies from one longitudinal girder to the next in a bay. It is sufficient for design purposes to record with a set of primary actions the most severe set of secondary loads occurring in each bay.

§6.2. *Case 2*

When the transport load is not on board, the remaining weights are balanced about the constant centre of gravity position, and each gasbag contains the same percentage of its maximum gas capacity, giving a total lift equal to the weight on board.

The procedure is then exactly similar to that described by Case 1, and the secondary loads must be found in the same way

§6.3. *Cases 3 and 4*

The cases for the light conditions, with and without transport load, are developed in the same way.

[14] See "Some Modern Developments in Rigid Airship Construction," by Lieut.-Colonel V. C. Richmond, Proc. Inst. Naval Architects, March 30th, 1928.

§6.4. *Case 5*

It has been pointed out that uniform deflation in the heavy condition[15] can occur as the result of superheating, and it is reasonable to assume that the maximum possible uniform deflation is equal, in tons of lift, to the lift of the gas lost in this way. The degree of superheating that is likely to occur depends on the size, the height, and the airspeed of the ship.[16] For a 5,000,000 cu. ft. airship, a figure of 10°C. may be assumed,[17] and the loss of lift may be computed very closely.[16] Working approximately, the loss due to 10°C. superheat is given by

$$(59/1000)\,(v_2 - v_1) = (59/1000)\,\{\, v_2 - (276/286)\, v_2 \,\} = (59/1000)\,(10/286)\, v_2$$

assuming that 10° superheat occurs at 2,000 metres, where the temperature is normally supposed 276°C., and that the corresponding lift of hydrogen is 59 lbs. per 1,000 cu. ft. v_2 is the maximum gas content, and v_1 the volume before superheating.

There will be found to be many ways in which weight may be dropped to provide equilibrium in this case, and the way chosen need not be that giving the worst shears and bending moments in any particular bay; rather should it be a way which the captain of the ship would be likely to use. The operation may need the movement of weights in the ship as well as the dropping of ballast, to provide a weight system in balance about the fixed centre of gravity.

When the weight distribution, and a new gas lift distribution consequent on the same percentage loss from each gasbag, have been determined, the procedure follows that described in detail for Case 1.

§6.5. *Cases 6, 7 and 8*

It was mentioned in §1.22 that the effect of uniform deflation is produced if the ship enters a region of warm air. The amount of this deflation may be taken for Cases 6, 7 and 8 as equal to the false lift that could result from the superheating of the gas contained in the ship in each of these cases. The procedure is then as has already been explained.

§6.6. *Cases 9-16*

When in any one of the foregoing Cases 1-8, the gasbag is considered completely deflated, ballast will be dropped. With the twofold object of preventing too serious local differences between weight and buoyancy, and of preserving the trim of the ship with the minimum movement of the remaining disposable weight, the ship's captain would in these circumstances drop weight from points near the deflated bag. This rule may be observed in arranging the case on paper, and in general it will be necessary to move ballast, and perhaps fuel, along the ship to keep it trimmed horizontally. Worse shears can be obtained by dropping ballast from points not so near the deflated bag, but it seems unnecessary to evolve impracticable cases; it is more difficult to frame a rule to decide what ballast should be supposed dropped in a given case, and some rule is desirable to give consistency to the sets of actions resulting from the series of deflations. The decision in this matter rests however with the designer; he can decide on the relative merits for design purposes of probable actions, and more severe but less probable actions. In either case, once the weights and lifts provide a system in complete equilibrium with the ship horizontal, the procedure of Table 4 is followed to obtain shearing forces and bending moments.

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[15] §1.22.

[16] "A Detailed Consideration of the Effects of Meteorological Conditions on Airships," by Major Scott and Lieut.-Colonel Richmond, Journal of the Royal Aeronautical Society, Vol. XXVIII., p. 189.

[17] *loc. cit.* Table 13.

§7. Details of Component A for Unpitched and Pitched Flight with no Yaw

§7.1. *Experimental Data Required*

The cases 17-52 cannot be developed without knowledge of the appropriate distributions of aerodynamic forces. The primary requirements for construction of the curves representing this loading are model tests providing tables of longitudinal force mX_θ, normal force mZ_θ, and pitching moment BM_θ for

(*a*) The bare hull over a range of angles of pitch of say $-20°$ to $+20°$.

(*b*) The hull with tail surfaces in position, rudders amidships, with elevators at different angles—say $0°$, $\pm 5°$, $\pm 10°$, $\pm 20°$, $\pm 30°$—over the same range of pitch angles at each elevator angle.

(*c*) The complete ship with flaps amidships, over the same range of pitch angles.

In addition to these tables it is very useful to have others giving the distribution of pressures over the hull for various pitch angles, though it will be seen[18] that these may be dispensed with, with probably no loss in accuracy in the determination of primary actions.

§7.2. *Curves of Normal Force Distribution*

The first convenient step is to develop diagrams representing the distribution of normal force on the bare hull for various degrees of pitch. The angles which may be chosen are $2°$, $5°$, $10°$, $15°$, $20°$, and if the distribution for any other angle in the range is required, it may be obtained by interpolation.

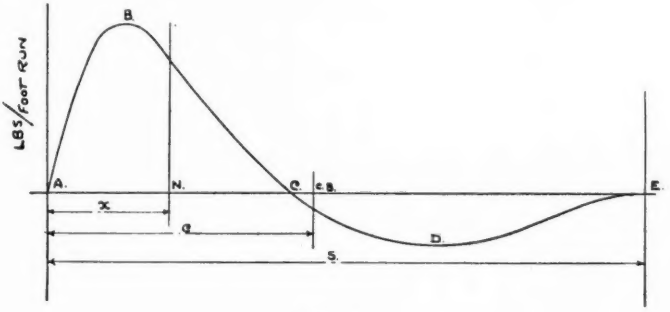

Fig. 5.

The "theoretical" method of determining the pressure distribution to be discussed, depends for its success on the ship having the front portion approximating to an ellipse in profile for some considerable distance. The profile of the airship R.33 from the nose to the beginning of the parallel portion is sufficiently close in shape to an ellipse, and represents a sufficient proportion of the ship's length to allow the "theoretical" method to be applied with considerable confidence in the approximation to the truth of the results. This confidence is strengthened in the cases of shapes such as those of R.100 and R.101, wherein the hulls from the noses to the maximum diameters are, almost exactly, hemispheroids extending for about 0.4 of the ships' lengths.

The method is based on the solution in pure hydrodynamics of the motion of a spheroid through a perfect fluid. Dr. R. Jones has developed this work, and

[18] *cf.* §§7.2, 7.5.

has supplied[19] formulæ defining the pressure distribution. The formula defining the distribution of normal force for pitch is

$$P = \tfrac{1}{2}\rho V^2 \left[\{ \pi (b^2/a) \sin\psi \cos\psi \} / \{ \sin^2\psi + (b^2/a^2)\cos^2\psi \} \right] (1+L)(1+M)\sin 2\theta \qquad (1)$$

where

$$L = -\{ (1/2e)\log(1+e)/(1-e) - 1 \} / \{ (1/2e)\log(1+e)/(1-e) - 1/(1-e^2) \} \qquad (2a)$$

and

$$M = -\{ \tfrac{1}{2}\log(1+e)/(1-e) - e/(1-e^2) \} / \{ \tfrac{1}{2}\log(1+e)/(1-e) - (e-2e^2)/(1-e^2) \} \qquad (2b)$$

e is the eccentricity of the elliptic profile, whose semi-major axis and semi-minor axis are a and b respectively. ψ is the eccentric angle of this ellipse, and θ is the angle of pitch; V is the speed of advance and ρ the density of air.

In applying this work to the airship, the assumption is made that this distribution is a first approximation to the distribution over the spheroidal bow portion of the ship; the ellipse approximating to this portion of the ship's profile is found, and from its geometry the distribution is worked out for $V=1$ f.p.s. from the formula above. This gives a curve somewhat as ABC in Fig. 5.

Normal force distributions for previous airships indicate that the shape of the remainder of the curve is somewhat as shown in the figure for CDE. A shape for this portion which may actually be adopted is given in Fig. 7. The curves ABC and CDE having been drawn in this way, the latter with no particular reference to vertical scale, it is considered that no more than the *shapes* of two portions of a complete curve ABCDE have been obtained, and discontinuity in slope at C is at this stage permissible. If the area ABEA above the datum represents n_a lbs., and the area CDEC below the datum represents n_b lbs., then it is suggested that

$$kn_a - ln_b \equiv (mZ_\theta)_H^{V=1} \qquad (3)$$

where $(mZ_\theta)_H^{V=1}$ is the total normal force, at $\theta°$ pitch, on the *hull* of the ship at 1 f.p.s. deduced from the tables mentioned in §7.1, and k and l are factors to be determined. It may reasonably be suggested that the neglect of viscosity in the hydrodynamical theory and the flat sides of the model necessitate the use of the factor k.

It is known that the pressure distribution obtaining at $\theta°$ pitch provides at 1 f.p.s. a normal force distribution curve whose total area represents $(mZ_\theta)_H^{V=1}$; the same pressure distribution provides a pitching moment $(BM_\theta)_H^{V=1}$. This moment, however, is *not* the moment of the diagram of Fig. 5, even when k and l are known, as the longitudinal components of the distributed pressures provide some part of the pitching moment. The pitching moment corresponding to the normal force distribution of Fig. 5 is obtained by plotting the ordinates in this diagram of points distant "d" from the centre of buoyancy, on a base of $\frac{1}{2}(d^2+y^2)$ where y is the radius of the ship at these points.[20] The reason for this is made clear in Appendix II. If AE is supposed to be an x-axis with origin at A, and if the C.B. occurs at $x=c$, the ordinate at x is plotted in the moment diagram at $\frac{1}{2}\{(c-x)^2+y^2\}$ producing the diagram of Fig. 6. If in this diagram the area ABCA above the datum represents m_a lbs. feet, and the area CDEA below the datum represents m_b lbs. feet, it is necessary that

$$km_a + lm_b = (BM_\theta)_H^{V=1} \qquad (4)$$

The equations (3) and (4) may be solved simultaneously to find k and l. These factors are then applied to their respective portions of the diagram of Fig. 5 and provide a normal force distribution diagram which agrees with the

[19] In R. & M. 600, R. & M. 780, and in "The Distribution of Normal Pressures on a Prolate Spheroid," Trans. Royal Society of London, Series A, Vol. 226, pp. 231-268.

[20] Trans. Roy. Soc., *loc. cit.*, p. 239.

experimental values obtained in the wind tunnel. The final curve may be discontinuous in slope at C, but, especially if this point is quite close to the C.B., as in modern shapes it is, the curve may be smoothed if desired without seriously affecting its area or its moment area. It represents the normal force distribution on the hull only at 1 f.p.s. For any other speed V, the ordinates must be multiplied by V^2.

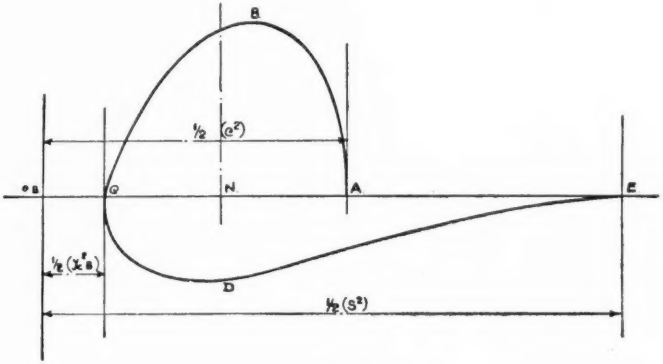

FIG. 6.

The procedure may be followed for the pitch angles suggested and the results plotted as a family of curves as shown in Fig. 7, which may be cross-plotted for interpolation purposes.

§7.3. *Curves of Variation of Shearing Force*

Another family of curves, representing the shearing forces due to these normal forces, may be obtained by integrating the curves of normal force distribution from the nose to the tail. Such curves are shown in Fig. 8; as no balancing forces are included, the end ordinates are not zero, but $(mZ_\theta)_H^{V=1}$ values for the hull at the various pitches.

§7.4. *Curves of Variation of Bending Moment*

The bending moment diagram for the aerodynamic forces at a given degree of pitch cannot be obtained by direct integration of the shear force diagram, for reasons apparent from Appendix II. The correct procedure is as follows:—

Consider in the typical shear force curve of Fig. 9 the point N, distant $(c-x)$ from the C.B.

Suppose N in Fig. 6 has the value of $\frac{1}{2}\{(c-x)^2+y^2\}$ corresponding to the value of N in Fig. 5. Then the bending moment at N, distant x from the nose is, as is shown in Appendix III., equivalent to

$$[\text{Area } NAB \text{ (Fig. 6)}] - (c-x)\ [\text{ordinate at } N \text{ (Fig. 9)}].$$

Taking N at points along the ship, the bending moment diagram results. The final ordinate is $(BM_\theta)_H^{V=1} + (mZ_\theta)_H^{V=1}(s-c)$, where s is the length of the ship. The family of bending moment curves corresponding with the loading and shear forces curves of Figs. 7 and 8 is given in Fig. 10.

§7.5. *Curves of Pressure Distribution*

If curves representing the pressure distribution have been obtained experimentally, it is possible to obtain from them curves of normal force distribution.

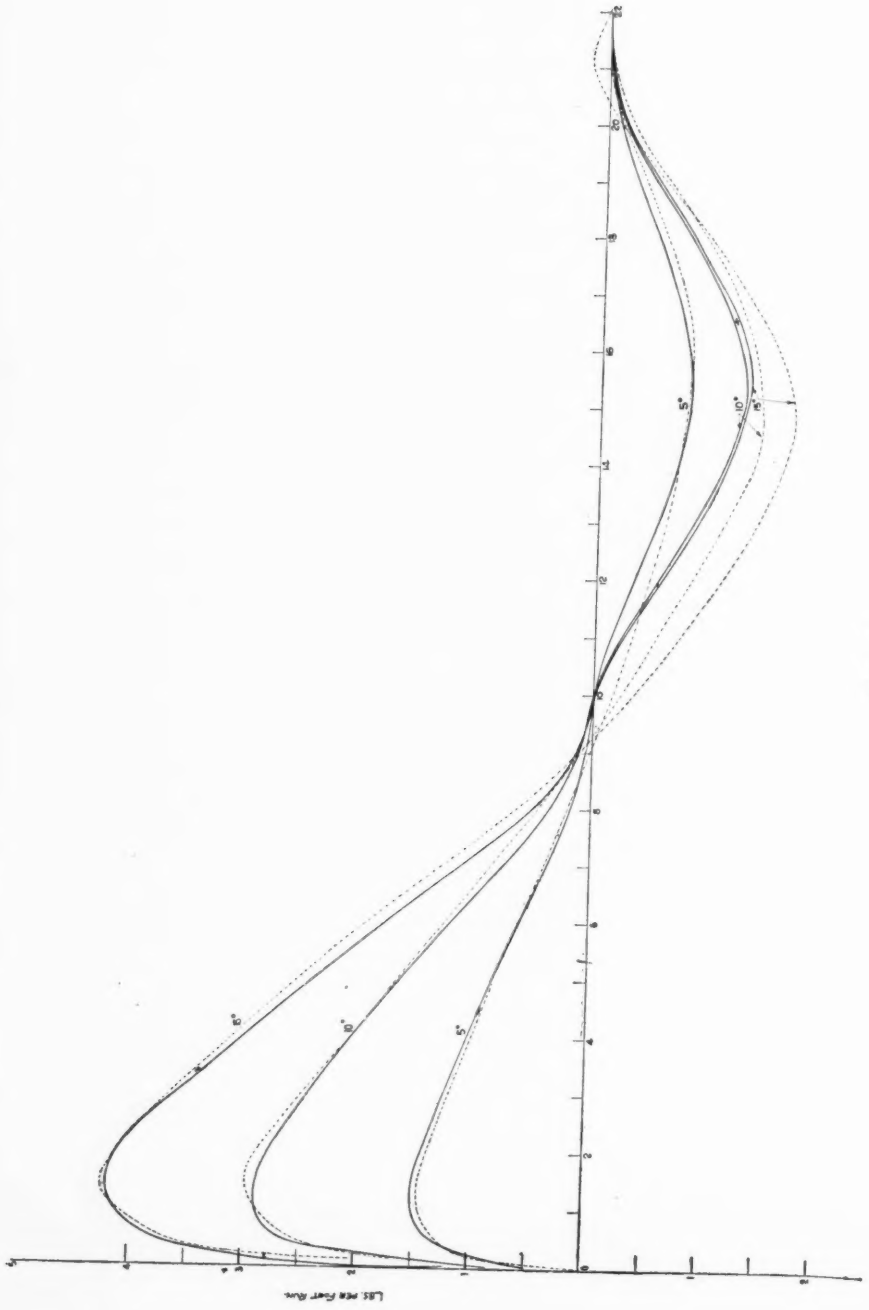

FIG. 7.
Aerodynamic Normal Force Distribution. At 1 ft. per sec. for 5°, 10° and 15° Pitch.

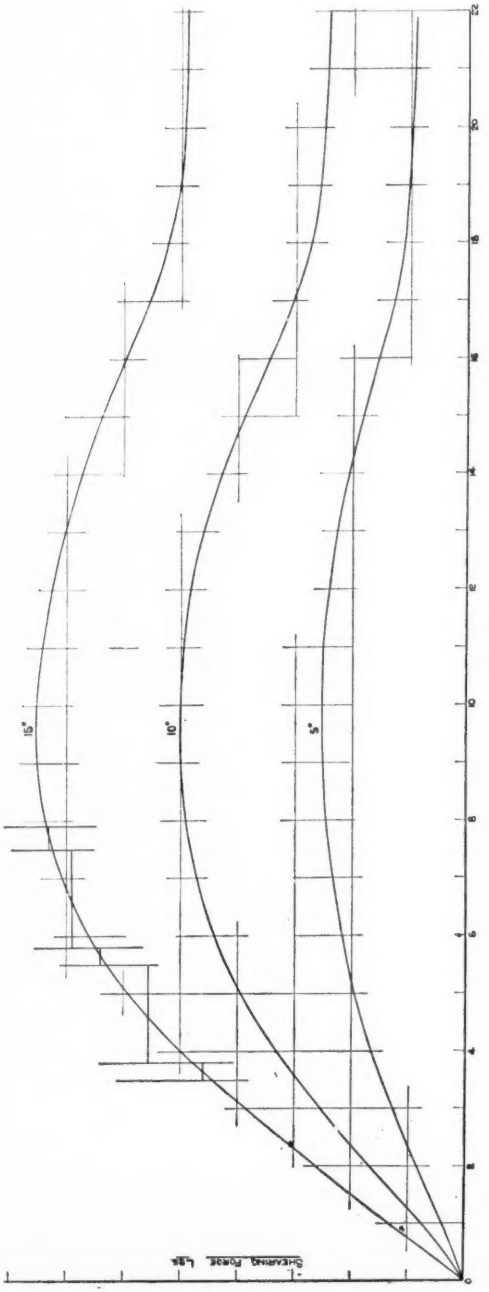

FIG. 8.
Shearing Force Diagram from Aerodynamic Normal Force Distribution for V = 1 foot per sec.

FIG. 7.
Aerodynamic Normal Force Distribution. At 1 ft. per sec. for 5°, 10° and 15° Pitch.

Experimental pressure distribution curves are given in Fig. 11 for longitudinal generators at angular intervals of 30°. If the pressure at any point on the hull is p lbs. per square foot, and if w is the angular co-ordinate of the point in a plane perpendicular to the ship's axis, then the total normal force on a section of the ship contained between two planes perpendicular to the axis and one foot apart is

$$P_y = r\int_0^{2\pi} p \cos w dw \qquad . \quad . \quad . \quad . \quad (1)$$

where r is the mean radius of the section in feet. Taking positions on the ship's axis at frequent intervals, this integral can be evaluated graphically at each and curves of normal force distribution result. On comparing the forces represented by the total areas of these curves, given by $\int_{\text{Nose}}^{\text{Tail}} P_y dx$, with the values of normal force $(mZ_\theta)_H$ given by the tables for the corresponding degrees of pitch, a difference is noted. Even when both pressure and plotting and total force measurement have both been made for a model of circular section the

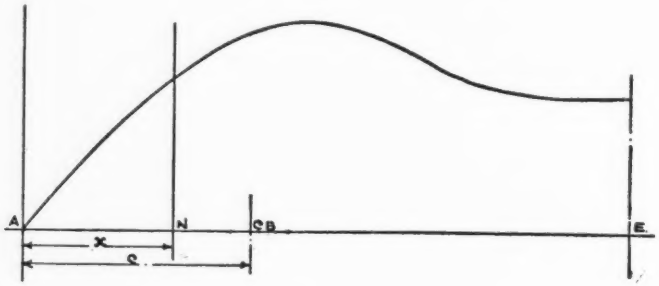

FIG. 9.

discrepancy may be as much as 30 per cent. at small angles, though the agreement of the pitching moments $(BM_\theta)_H$ found by integration and by direct measurement is often extremely good. As an illustration of the order of agreement, Table 5, which refers to measurements on a model of R.101, is reproduced from a paper by Lieut.-Colonel Richmond.[21]

TABLE 5.

Pitch.	Integrated value of $\frac{(mX_\theta)_H}{(mX_\theta)_H}$ measured on balance.	Integrated value of $\frac{(mZ_\theta)_H}{(mZ_\theta)_H}$ measured on balance.	Integrated value of $\frac{(BM_\theta)_H}{(BM_\theta)_H}$ measured on balance.
0	·395	—	—
5	·59	·71	·92
10	—	·99	·99
15	·71	1.04	·97
20	—	1.10	·97

[21] "A Review of the Present Position with Regard to Airship Research and Experiment," Journal of the Royal Aeronautical Society, Vol. XXX., p. 552.

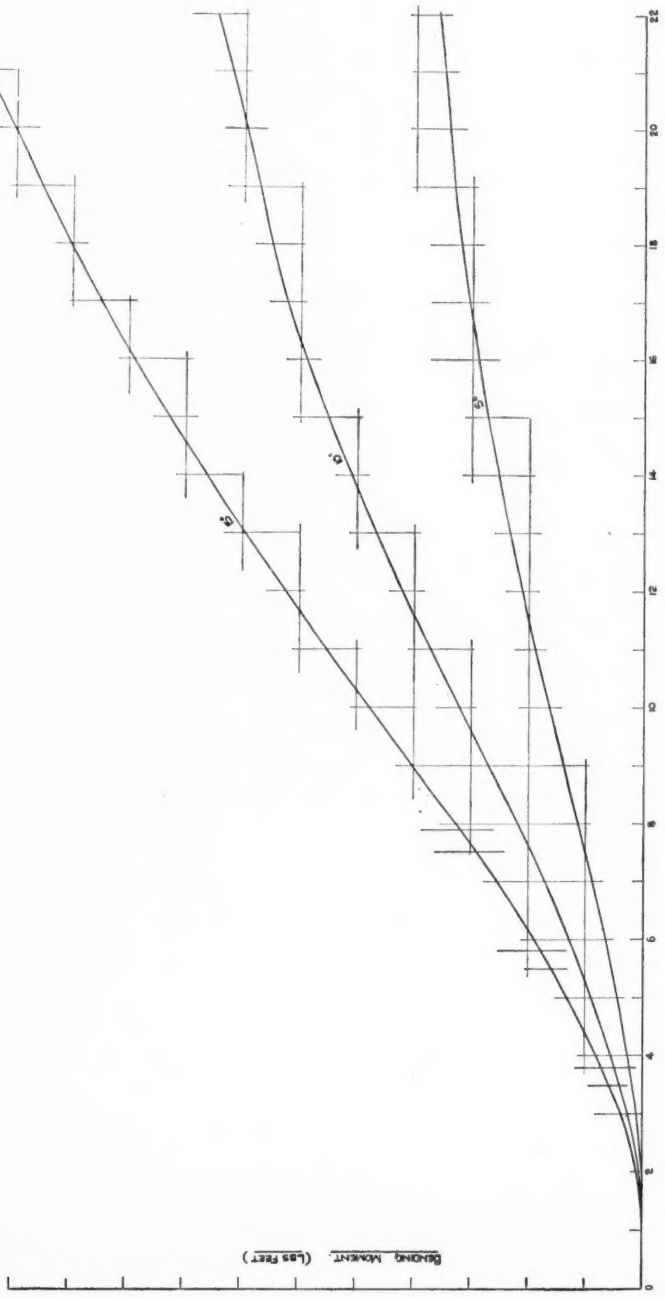

FIG. 10.

Bending Moment Diagram from Aerodynamic Normal Force Distribution for V=1 foot per sec.

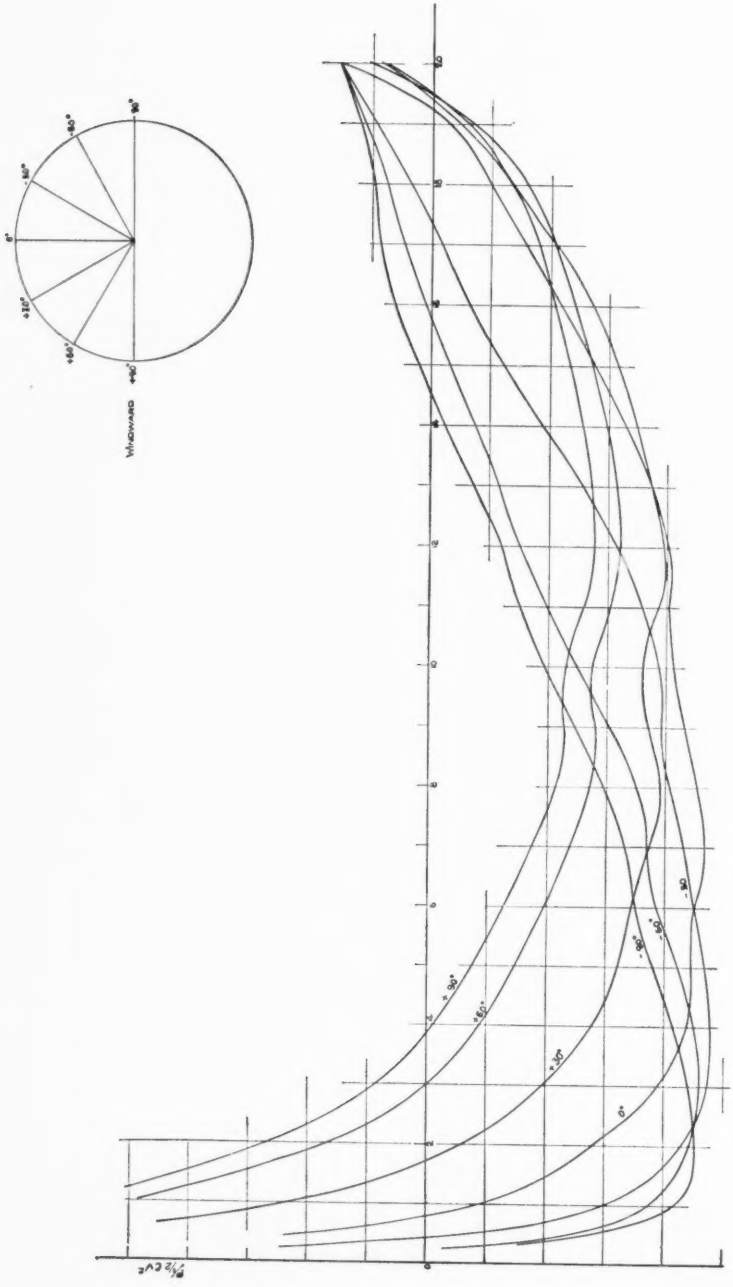

FIG. 11.

It is supposed that the discrepancies are due to skin frictional forces, which are automatically embraced in force and moment measurement on the balance, but are not recorded in pressure plotting. The advantages of the theoretically determined normal force distribution diagram (§7.2), from the point of view of computation begin to be apparent; it takes account of the effects of viscosity by means of the balancing factors k and l—though perhaps not in the way that research will show at some future time to be a physically correct way—and it also includes the effect of the flattened sides of the airship on the total forces and moments.

The increment of longitudinal force corresponding with the normal force increment of equation (1) is

$$P_x = r\,(dr/dx)\int_0^{2\pi} p\,dw \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where dr/dx is the mean slope of the shape at the section considered. From the experimental pressure distribution curves, curves of distribution of longitudinal force may be found graphically, and their areas, represented by $\int_{\text{Nose}}^{\text{Tail}} P_x dx$ give the total longitudinal forces they account for. Comparing these forces with $(mX_\theta)_H$ from the tables of direct force measurement, discrepancies such as are given in Table 5 are found.

In Fig. 7 curves of normal force distribution found by integration of pressure plotting results for a model of circular section are shown dotted. The full curves are the "theoretical" ones for the same model when made polygonal in section. The differences between corresponding full and dotted curves may therefore be supposed to indicate the effects of skin friction and flat sides. If "theoretical" diagrams are built up from a knowledge of forces and moments on the circular section model, it is possible to separate the two effects, though the results are not particularly trustworthy. It is probably safer to argue from the figures in Table 5. The factor k introduced the *normal force* effects of skin friction and flat sides as a linear function of P_y. If it be assumed that the skin friction effect alone is a linear function of P_y, then the shape of skin friction normal force diagram giving the Table 5 requirement of net force but no net moment is as in Fig. 12*a*.

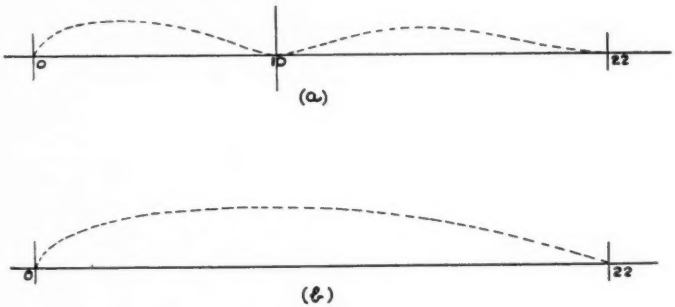

FIG. 12.

If such indefensible assumptions are accepted for the moment, then the longitudinal distribution of frictional forces may be supposed obtained from

this figure by multiplying it by a curve of dx/dy for the ship's shape.[22] The result of this is to produce a shape somewhat as in Fig. 12*b*. Thus, in attempting to find a longitudinal distribution of frictional force consistent with the implications in the use of k, a distribution not unlike the shape of the ship is arrived at. If again it is considered that the frictional forces are proportional to the distribution of surface area, then the result is a curve of $y\ (dx/ds)$ for the ship's profile to represent the longitudinal distribution. Obtaining curves by both rough arguments something like the ship's shape, leads to the suggestion that, until correct methods are known, the distribution of skin frictional forces may be represented by diagrams of the ship's shape with ordinates multiplied so that their total areas make up the deficiencies in drag such as are exhibited in Table 5.

Curves of pressure distribution are necessary for the determination of the secondary loading of girders due to aerodynamic loads transmitted to them by the cover and for the primary longitudinal forces, so that if experimental pressure plotting is not available, some attempt at developing the probable distributions at different pitch angles is required. Such an attempt can be made with the help of the hydrodynamical work on the spheroid due to Dr. Jones. Referring to this, it is found[23] that the theoretical pressure at any point on the spheroid is given by

$$p = a_0 + a_1 \cos w + a_2 \cos^2 w + a_3 \sin^2 w \quad . \quad . \quad . \quad (3)$$

where a_0, a_1, a_2 and a_3 are given in Appendix III., and w is the angular coordinate of the point in the plane containing the point, normal to the major axis.

It is clear that the lateral force, as given by equation (1), §7.2, is the result of performing the operation $r\int_0^{2\pi} p \cos w dw = b \sin \psi \int_0^{2\pi} a_1 \cos^2 w dw$. To obtain a pressure distribution over the spheroidal nose of an airship corresponding with the normal force distribution obtained in §7.2, it is necessary, using the formula (3), to plot $p' = kp$. This may be done for longitudinal generators denoted by $w = 0°$, $\pm 30°$, $\pm 60°$, $\pm 90°$, until the end of the spheroidal portion is reached; the portion of the curves in Fig. 13, between the vertical axis and the line AB, represents this step. The continuation of the curves can only be achieved by trial and error. By cross-plotting the portions of the curves so far obtained, the manner in which p' varies *round* the ship for sections successively further from the nose can be appreciated. Such curves are shown in Fig. 14. The curves in Fig. 13 are continued tentatively on to some line CD and the resulting values plotted in Fig. 14. Then by plotting $p' \cos w$ the quantity $r\int_0^{2\pi} p' \cos w dw$ can be obtained. By continuing the Fig. 13 curves in a natural way with some knowledge of how such curves may be expected to run, and

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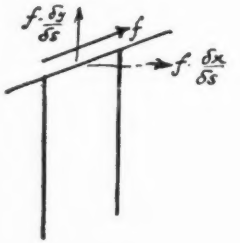
[22] If the skin friction force f on an element is supposed tangential to the surface there, the normal component will be $f(\delta y/\delta s)$ and the longitudinal component will be $f(\delta x/\delta s)$



[23] *loc. cit.* §7 2.

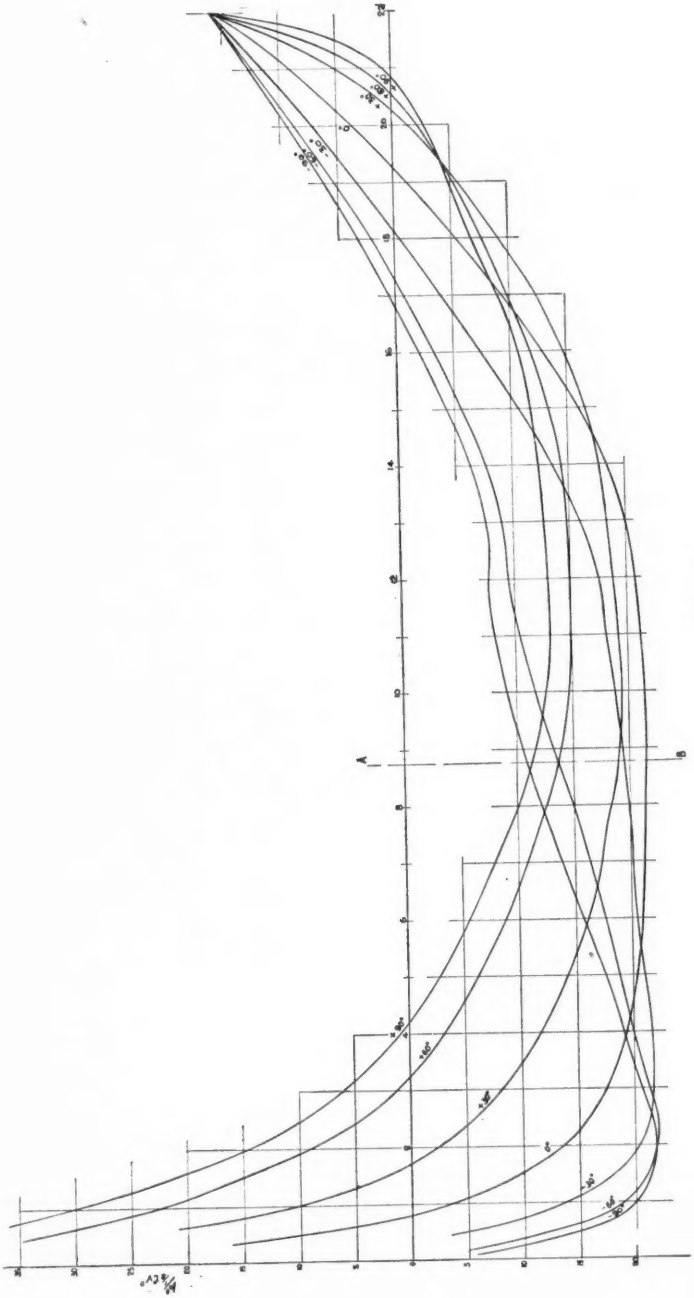


FIG. 13.

making any necessary modifications with the help of Fig. 14 and the fact that the quantity $r\int_0^{2\pi} p' \cos w dw$ must equal the corresponding ordinate of the normal force distribution diagram for the same degree of pitch in Fig. 7, a final set of fresh points on the curves in Fig. 13 may be obtained. It is quite easy to continue this process until the tail is reached, and the operator always feels that he has obtained the only curves that will agree with the normal force distribution already found. It is difficult to say whether this is so, but there certainly seems very little choice left to him in pursuing the method. In the examples which the writer has completed in this way and compared with the results of wind tunnel experiment, the agreement has been encouragingly close,

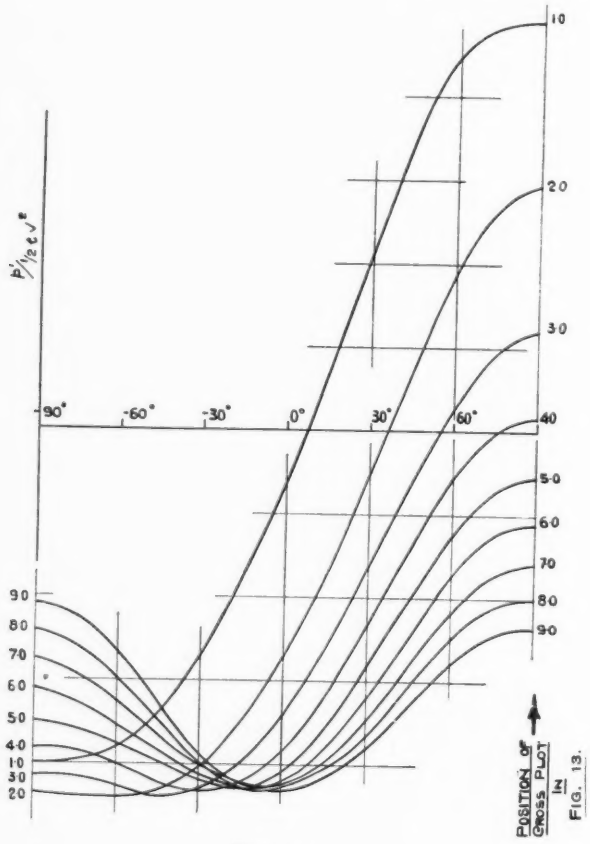


FIG. 14.

though it must be remembered that in using $p'=kp$ and forcing agreement with the curves of Fig. 7, some effects due to skin friction and the flat panels of the ship are introduced. Fig. 11 illustrating the results of pressure plotting in the wind tunnel, and Fig. 13 illustrating the synthetic method just described, for the same degree of pitch may be compared.

It is clear that for 0° pitch, as the normal force distribution diagram for the hull is everywhere zero, the method described cannot be applied to obtain the curves of Fig. 13, which then become coincident. Careful cross-plotting of the curves obtained for other degrees of pitch, however, allow extrapolation of sufficient accuracy.

The distributed loading on the ship's girders, due to aerodynamic forces, can be obtained from these curves. It depends on the system of attaching the cover, the initial tension in the cover, and on the possible degrees of pressure, above and below that of the atmosphere, which can occur within the ship. This last factor may be based on the results of rents in the cover in high and low pressure regions, but with the other factors just mentioned is too much the problem of a particular design to be dealt with here.

§7.6. *Curves of Longitudinal Force Distribution*

It has already been indicated that the distribution of longitudinal forces on the airship can be represented by plotting P_x (equation (2), §7.5). The form the resulting curves take is given in Fig. 15. The total longitudinal force existing at a point distant X ft. from the nose is given by $\int_{\text{Nose}}^{X} P_x dx$, and this integral is evaluated at frequent intervals along the ship's axis to give the distribution of total longitudinal force. This is done for the same degrees of pitch for which normal force distributions were found, and the result is a family of curves corresponding with those of Figs. 7, 8 and 9. Dimensionally, the curves correspond with those of Fig. 8; a typical example is given in Fig. 16.

The use of the artificial pressure distribution curves in this estimation of longitudinal forces does not mean that the final ordinate of curves such as that in Fig. 16 will be equal to the corresponding longitudinal force $(mX_\theta)_H$ from the experimental tables. The pressure curves must, in having been made to correspond with the normal force curves, contain what are in effect the normal components of the skin frictional forces, but the longitudinal components have still to be accounted for. They may be introduced as ordinates proportional to the integral curve of the diagram of Fig. 12, which may again be supposed to have the same shape as the hull profile, the scale being such that the final ordinate, together with the ordinate due to the pressure distribution, gives the correct total longitudinal force.

§7.7. *The Forces due to the Horizontal Fins and Elevators*

From the experimental tables (*a*) and (*b*) of §7.1, forces and moments due to the horizontal fins and elevators can be found. Integration of the results of pressure plotting experiments on fins and elevators prove that these forces and moments are not entirely due to the pressures on these surfaces, but partly to alterations they cause to the pressure distribution on the hull. The distribution of loading on these tail surfaces is generally somewhat as sketched in Fig. 17. For a given positive pitch, as the elevator angle increases positively or negatively from 0°, so does the centre of pressure of this distribution move aft or forward, and as the pitch increases with elevators amidships the C.P. moves forward. For the purposes of primary stressing it is safe to deal rather arbitrarily with these effects. Fig. 17 may be taken as representing the *shape* of the distribution of forces due to the tail surfaces, whatever the degree of pitch or elevator. For a given pitch and elevator angle, the normal force $(mZ_\theta)_F$ and pitching moment $(BM_\theta)_F$ due to the fins and elevators give the position of the C.P. of the forces due to them; its distance from the C.B. is $\lambda = (BM_\theta)_F/(mZ_\theta)_F$. The diagram of Fig. 17 can then be made to represent the

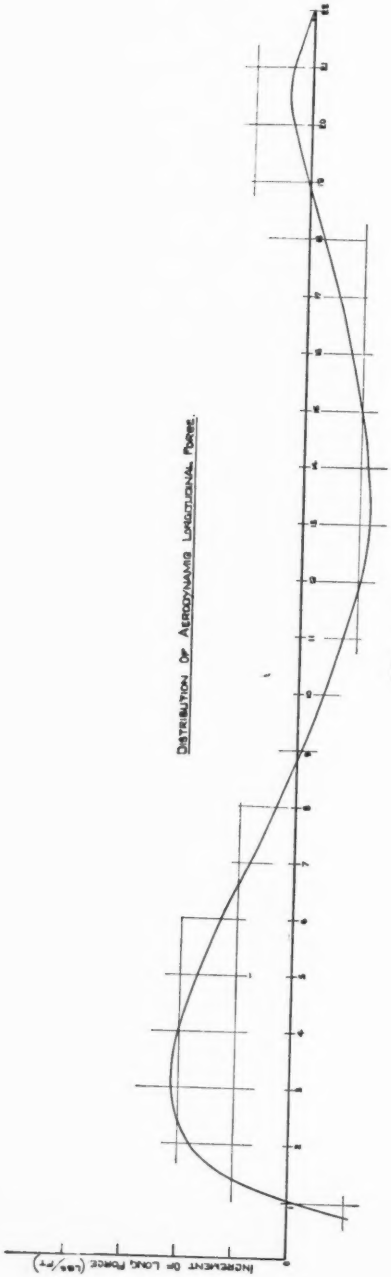

DISTRIBUTION OF AERODYNAMIC LONGITUDINAL FORCE.
INCREMENT OF LONG FORCE (lbs/ft)

FIG. 15.

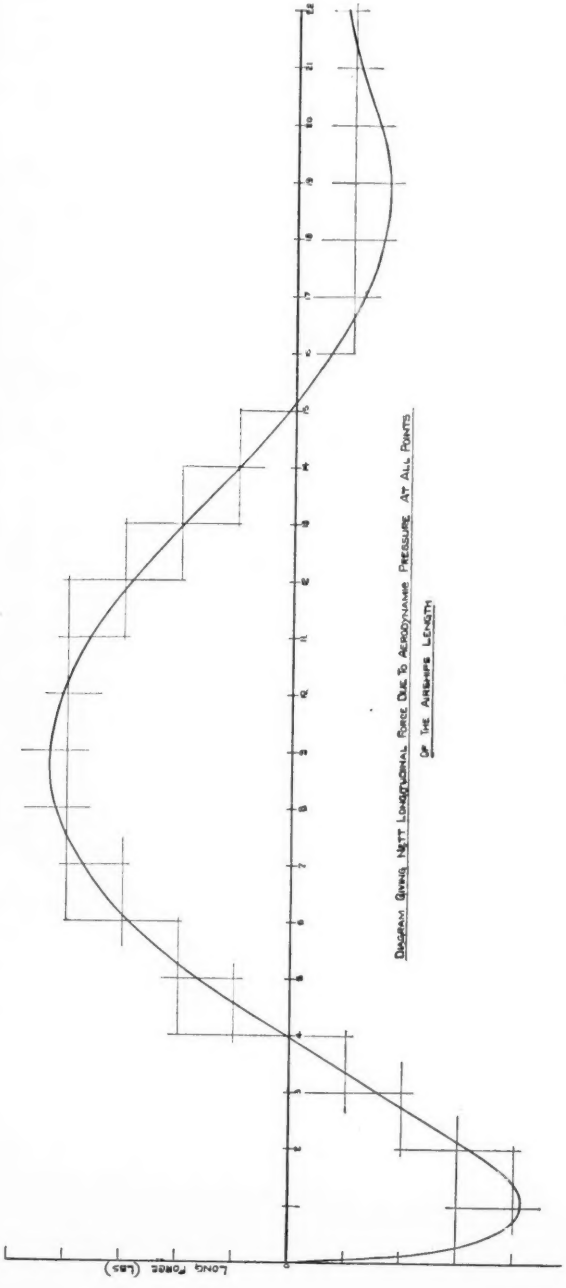


Diagram Giving Nett Longitudinal Force Due to Aerodynamic Pressure at All Points of the Airships Length
Long Force (Lbs)

FIG. 16.

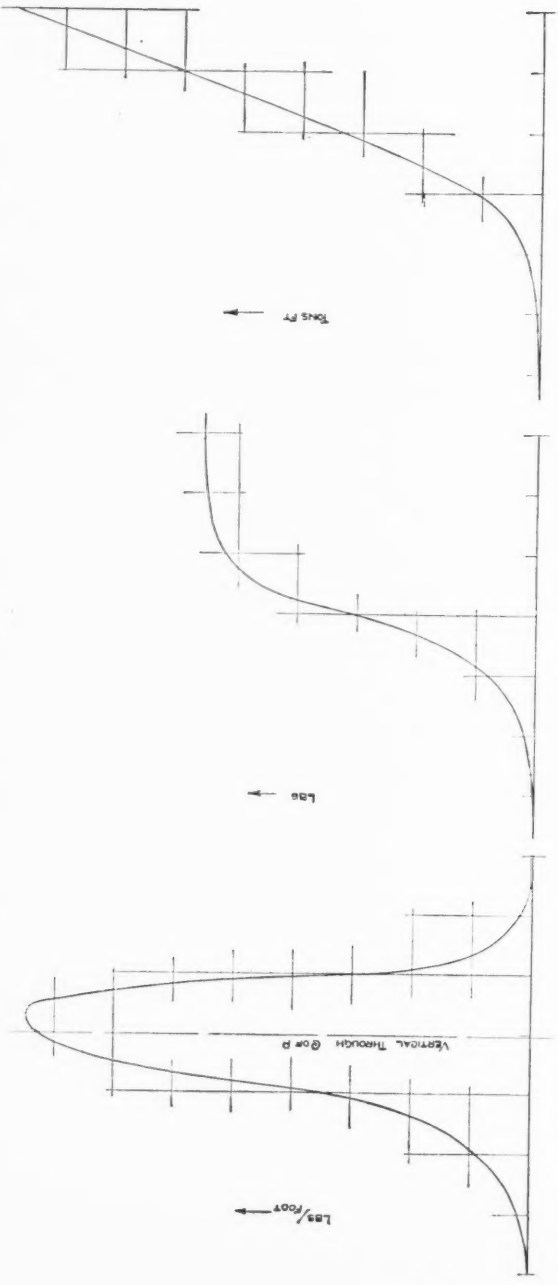


FIG. 17.
Typical Distribution of Aerodynamic Load on Fins and Elevators.

FIG. 18.
Aerodynamic Shear Force Diagram for Fins and Elevators for Load Dist. of Fig. 17.

FIG. 19.
Aerodynamic Bending Moment Diagram for Fins and Elevators for Load Dist. of Fig. 17.

conditions by multiplying its ordinates to give a total area $(mZ_\theta)_F$ and by moving it bodily forward or aft until its C.P. is λ ft. from the C.B. of the ship.

The adoption of this standard gives a fixed shape for the shearing force diagram, shown in Fig. 18. From its final abscissa to the tail, the shear is constant and the ratio of $(mZ_\theta)_F$ to this shear gives the factor by which the shear diagram must be multiplied to fit a particular case. The standard diagram is consistent with a definite C.P. position λ_1 and the movement of this shear diagram forward or aft is given by $\lambda_1 - \lambda$.

The bending moment diagram, shown in Fig. 19, is multiplied by the same factor and moved in the same way. The extent of the final uniformly increasing portion varies in consequence, but introduces no difficulty.

The secondary loading on the fins and moving surfaces is a matter that cannot be settled very satisfactorily without recourse to pressure experiments. Comparison with previous experiments of this kind on model and full-scale is the next best course, but no satisfactory empirical methods, such as have been described for the aerodynamic forces on the hull, can be suggested at present.

The longitudinal forces due to the fins and elevators are not very important, and may be presumed to be distributed over the fins and elevators as if all equal elements of area contributed equal increments. This means that the total longitudinal force from this source up to any point will be proportional to the total fin area ahead of an ordinate at that point.

§7.8. *The Forces due to the Cars*

Unless external cars are very large, it will be deduced from the experimental tables that their effect on normal force is negligible. Their longitudinal forces $(mX_\theta)_C$ are quite important however, but may be considered to be applied to the main structure at the points of attachment of the cars, no attempt to allow for interference effects being made.

§7.9. *"Stepping" of Diagrams*

It was pointed out in §4 that the shear across a bay or a frame is represented diagrammatically by a horizontal straight line, and the bending moment by a sloping straight line. The diagrams obtained by the methods of this §7 have been drawn without reference to the position of frames, however, and need adapting. If the positions of frame faces are indicated by ordinates, as in Fig. 8, then the stepping is carried out by drawing horizontal lines at such heights that the area of shear diagram contained between adjacent ordinates is unaltered. Consistent with this, in Fig. 9, only the bending moment values at frame faces are recognised, and the points marking these may be joined by straight lines.

§8. Details of Component T

In the absence of precise data for the particular engines and propellers proposed for the ship for which external load systems are required, a good approximation to the variation of thrust with speed, some knowledge which is necessary, may be deduced from the most appropriate curve among those given on p. 144 et seq. of Dr. Watts' "Screw Propellers for Aircraft." A typical example which can be applied to the airship is taken from p. 147 and given in Fig. 20. As *overall efficiency* is proportional to thrust, if the speed ordinates of this curve are divided by their abscissæ a curve which to some scale gives the variation of thrust with speed can be drawn as in Fig. 21. The portion of this curve which is likely to be used in external load investigations can, for simplicity, be replaced by a straight line as shown in the figure. This straight line may be designated

$$y = mx + c$$

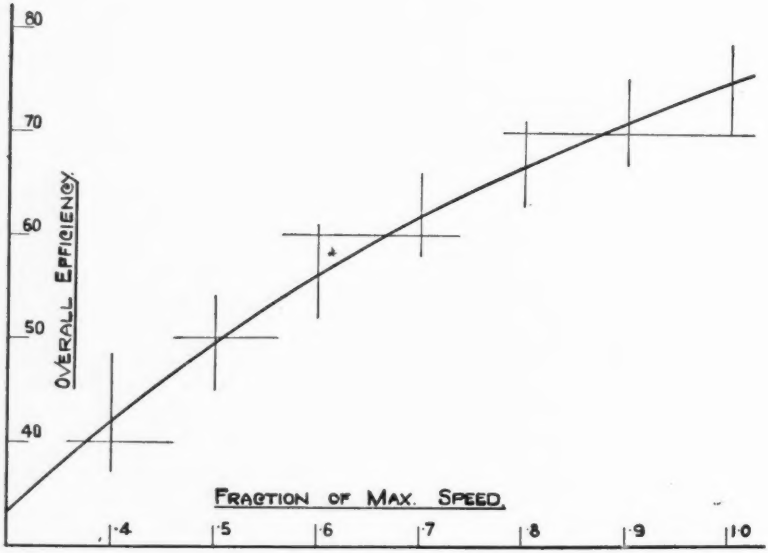

80
70
60
50
40
OVERALL EFFICIENCY
FRACTION OF MAX. SPEED.
·4
·5
·6
·7
·8
·9
1·0

FIG. 20.

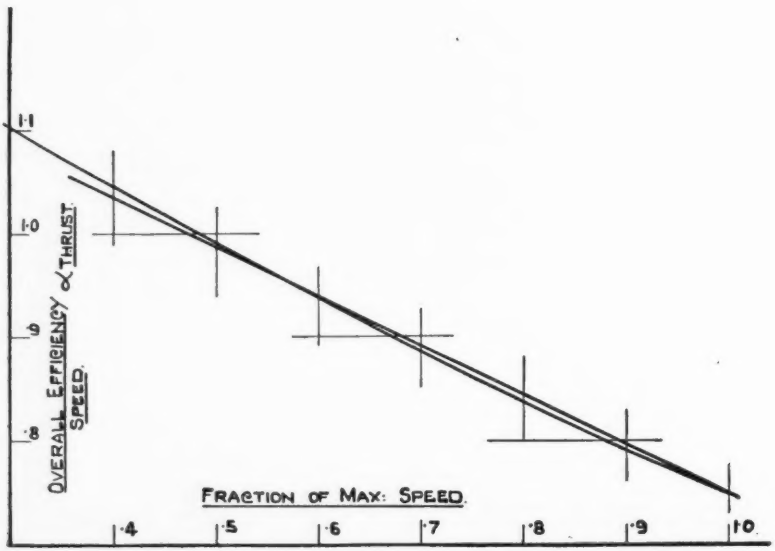

1·1
1·0
·9
·8
OVERALL EFFICIENCY OF THRUST
SPEED.
FRACTION OF MAX: SPEED.
·4
·5
·6
·7
·8
·9
1·0

FIG. 21.

where m and c are straightway determinable, and x = speed/ max. speed. Then

$$\text{Thrust} = jy = j\,(mx + c) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When the desired maximum speed of the ship has been fixed, the head resistance at maximum speed can be deduced from the experimental tables, and then

$$V^2_{\max}\,(mX)^{V=1}_{\substack{\text{Complete}\\\text{Ship}}} = j\,(mx + c) \quad . \quad . \quad . \quad (2)$$

where $(mX)^{V=1}_{\substack{\text{Complete}\\\text{Ship}}}$ represents the drag of the whole ship at o° pitch and o° yaw; j may therefore be determined and hence (1) becomes the law of variation of thrust with speed. It may be more conveniently written, after substitution from (2)

$$T = K - kV \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where K and k are constants.

In all airships so far constructed, the line of action of the thrust has been below the ship's axis. A mean line of action can be calculated from the powers and positions of the various airscrews, but it is doubtful whether the thrust acts along such a line as the proximity of the hull probably deflects the propeller airstreams.

Full-scale experiment could throw some light on the problem; it would appear that attempts could be made to measure the pitch caused by the thrust, and so provide the means of finding its mean line of action. In the absence of reliable data, there seems no alternative to supposing the mean line to be that given by taking moments of the several thrusts about the C.B. and dividing by the total thrust.

§9. Cases 17-52 (Cases of Pitched and Unpitched Flight)

§9.1. *Nomenclature*

In discussing the equilibrium of the conditions represented by Cases 17-52, reference is made to Fig. 22, wherein

θ = angle of pitch, $+^{ve}$ as drawn.
α = angle of flight path, $+^{ve}$ as drawn.
ϵ = angle of elevators, $+^{ve}$ as drawn.
W = sum of static weights acting at the C.G., C_W.
L = total lift supposed acting at C.B., C_L.
(mX) = longitudinal force due to hull and fins at θ degrees of pitch and ϵ degrees of elevator at 1 f.p.s., $+^{ve}$ as drawn.
(mX_c) = longitudinal force due to cars or other external structure carrying airscrews at 1 f.p.s., $+^{ve}$ as drawn.
(mZ) = normal force due to hull and fins at θ degrees of pitch and ϵ degrees of elevator at 1 f.p.s., $+^{ve}$ as drawn.
M = static moment due to weights and lifts which the condition of θ° pitch and ϵ° elevator can balance, $+^{ve}$ as drawn.
T = thrust of propellers.
τ = perp. distance of net thrust line from ship's axis.
BM = pitching moment due to hull and fins at θ degrees of pitch and ϵ degrees of elevator at 1 f.p.s., $+^{ve}$ as drawn.
V = air speed along the flight path.
h_W = perp. distance of C_W from ship's axis.
h_L = perp. distance of C_L from ship's axis.

§9.2. *Cases 17-32*

In the cases where the ship is flying horizontally and unpitched, the thrust and cars are exercising a pitching moment. This must be balanced by moving weight forward.

The equations of equilibrium are then

$$(mX + mX_c)\, V^2 + T = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$W - L = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$mX_c V^2 \tau + T\tau + M = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where the thrust is defined as in §8, by $T = K - kV$.

When the moment M has been determined, and the movement of ballast has been defined, the new distribution can be written down as in column 3 of Table 4, and the shearing forces can be calculated as described in §6.1. This step corresponds with equation (2). Integrating the shearing forces, a column of bending moments due to this distribution can be found, but as it represents only the M term in equation (3) it will naturally not grow to zero at the tail. It is necessary to add algebraically to these bending moments, those due to the moment $(T + mX_c V^2)\, \tau$. If this moment is applied to frames of the ship, then it can be divided up between these frames according to the power and drag of the power units, and their distances from the ship's axis. An example illustrating this will be given in §9.36.

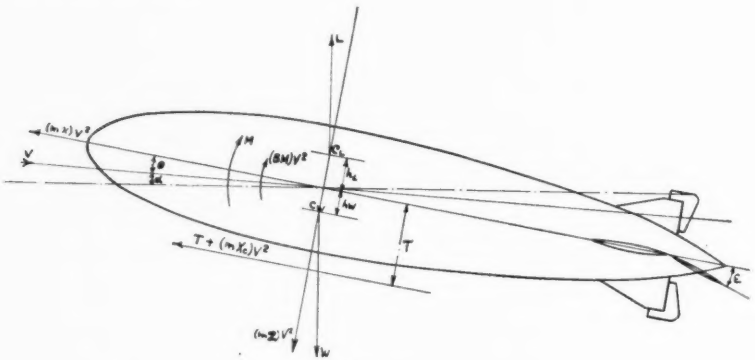


FIG. 22.

Longitudinal forces are represented by equation (1). The $(mX)\, V^2$ term follows from the stepped longitudinal force diagram for the hull at 0° pitch (§§7.5, 7.6) from the fin longitudinal force diagram and from the skin friction adjustment.

These forces are balanced by differences of thrust and car drag applied at certain frames. An example illustrating this is given in §9.37.

§9.3. *Cases 33-44*

In the cases where the ship is flying with positive pitch to balance the loss of static lift, the equations of equilibrium are

$$(mX + mX_c)\, V^2 + T - (W - L) \sin \alpha + \theta = 0 \quad . \quad . \quad . \quad . \quad (1)$$

$$(mZ)\, V^2 + (W - L) \cos \alpha + \theta = 0 \quad . \quad . \quad . \quad . \quad (2)$$

$$(BM_\theta)\, V^2 + mX_c V^2 \tau + T\tau - W h_W \sin \alpha + \theta - L h_L \sin \alpha + \theta + M = 0 \quad . \quad (3)$$

where the thrust is defined by

$$T = K - kV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The complete solution of these equations may be obtained in the following way for the possible ranges of pitch angles and elevator angles chosen.

§9.31. First of all, the value of W for the heavy or light condition which is being investigated is found; it will be the W value for one of the cases 1, 2, 3.

4, of Table 3. Then a value of θ is chosen; this may reasonably be one of the set 2°, 5°, 10°, 15°, suggested before (§7.2).

§9.311. Next, the value of ϵ at one end of the range of elevator angles, which may be $-20°$, $-15°$, $-10°$, $-5°$, 0°, 5°, 10°, 15°, 20°, is selected. Then for the θ and ϵ values chosen, (mX), (mZ), (BM), (mX_C), are written down.

The next step is to choose a value for α. A suitable value to begin the investigation with is $\alpha = 0°$. Then $\sin(\alpha+\theta)$ and $\cos\alpha+\theta$ are written. From equation (2), $(W-L)$ then appears in terms of V^2 only. Substituting this value of $(W-L)$ in equation (1) results in a quadratic in which V is the only unknown. When V is found, the value of $(W-L)$ follows immediately, and hence the value of L. By substituting in equation (3), M is then found.

Another value of α, say $\pm 5°$, is then selected, and the particular solution just described is repeated. Further selections for α are made to complete a suitable range, whose limits will be decided by what is considered to be the maximum allowable inclination $(\alpha+\theta)$ of the ship and the maximum $(W-L)$ value which can possibly occur in it.

§9.312. The next value in the ϵ range of angles is now taken, and the procedure of §9.311 is performed again, and is repeated for all the values in the ϵ range unless impossible conditions of α or $(W-L)$ occur.

§9.32. The other values in the θ range of angles are then dealt with in the same way as has been described for the first in §9.31 all for the same value of W.

For each of the remaining three W values the whole procedure is repeated.

§9.33. The complete solution is then best exhibited in the form of curves. For a given W there are required for each θ value, curves of:—

1. α plotted against $(W-L)$ for different ϵ values as in Fig. 23.
2. V plotted against $(W-L)$ different ϵ values as in Fig. 24.
3. $-M$ plotted against $(W-$, or different ϵ values as in Fig. 25.

§9.331. Another set of curves which will be required if this pitched flight investigation is continued on rather exact lines, can be drawn to show the variation with the ship's inclination of the moment due to the deflation of each of the different gasbags. It is a refinement that can conveniently be dispensed with, but if the very considerable labour involved in carrying out the procedure described in the preceding paragraphs has been completed, it represents but a small addition to that labour.

For each of the Cases 1-8, a curve can be drawn for each gasbag showing the variation of the moment due to its deflation as $(\alpha+\theta)$ varies. If the gas lift lost is f lbs., with its C.B. originally X feet along the axis from the ship's C.B., and H feet perpendicularly above the axis, the moment M is given by the following formulæ:—

Gasbags forward of C.B.

$$-M = f(X\cos\alpha+\theta - H\sin\alpha+\theta) \quad . \quad . \quad . \quad (5a)$$

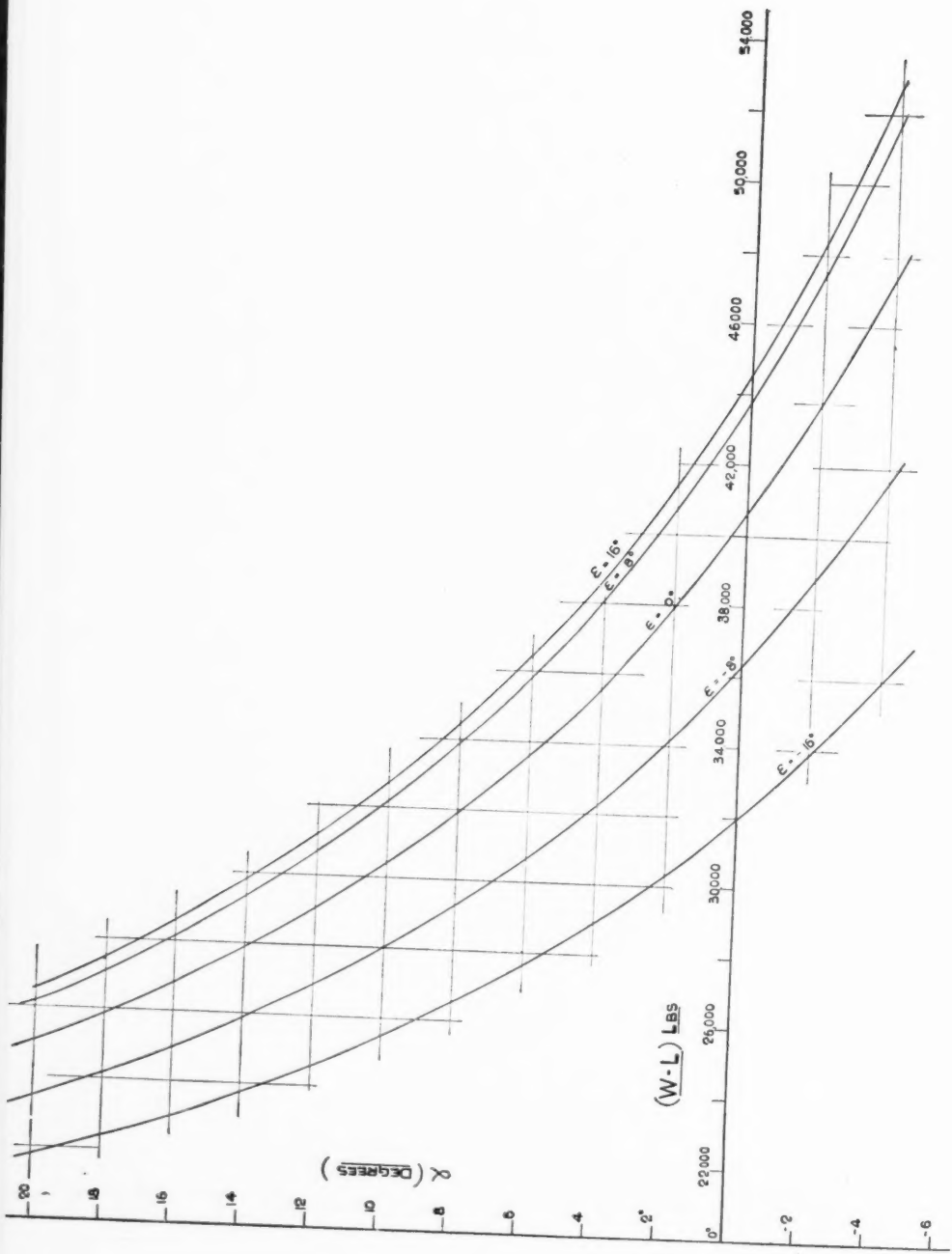
Gasbags aft of C.B.

$$-M = -f(X\cos\alpha+\theta + H\sin\alpha+\theta) \quad . \quad . \quad . \quad (5b)$$

Examples of such curves are shown in Fig. 27 marked (*a*).

§9.34. *Use of the Curves*

Now suppose that in, for example, Case 33 at 10° pitch, the gasbag deflated has a $(W-L)$ value of $(W-L)_p$. At this value of the abscissa on each of the diagrams of Figs. 23, 24, 25, an ordinate is drawn. Plotting values of the points where these ordinates cut the ϵ curves, the variations of M with $\alpha+\theta$ and of V and α with ϵ are obtained. Curves for these variations are shown in Figs. 26, 27, 28.


α (DEGREES)
(W-L) LBS
ε = 16°
ε = 8°
ε = 0°
ε = -8°
ε = -16°
20
18
16
14
12
10
8
6
4
2°
0°
-2
-4
-6
22000
26000
30000
34000
38000
42000
46000
50000
54000

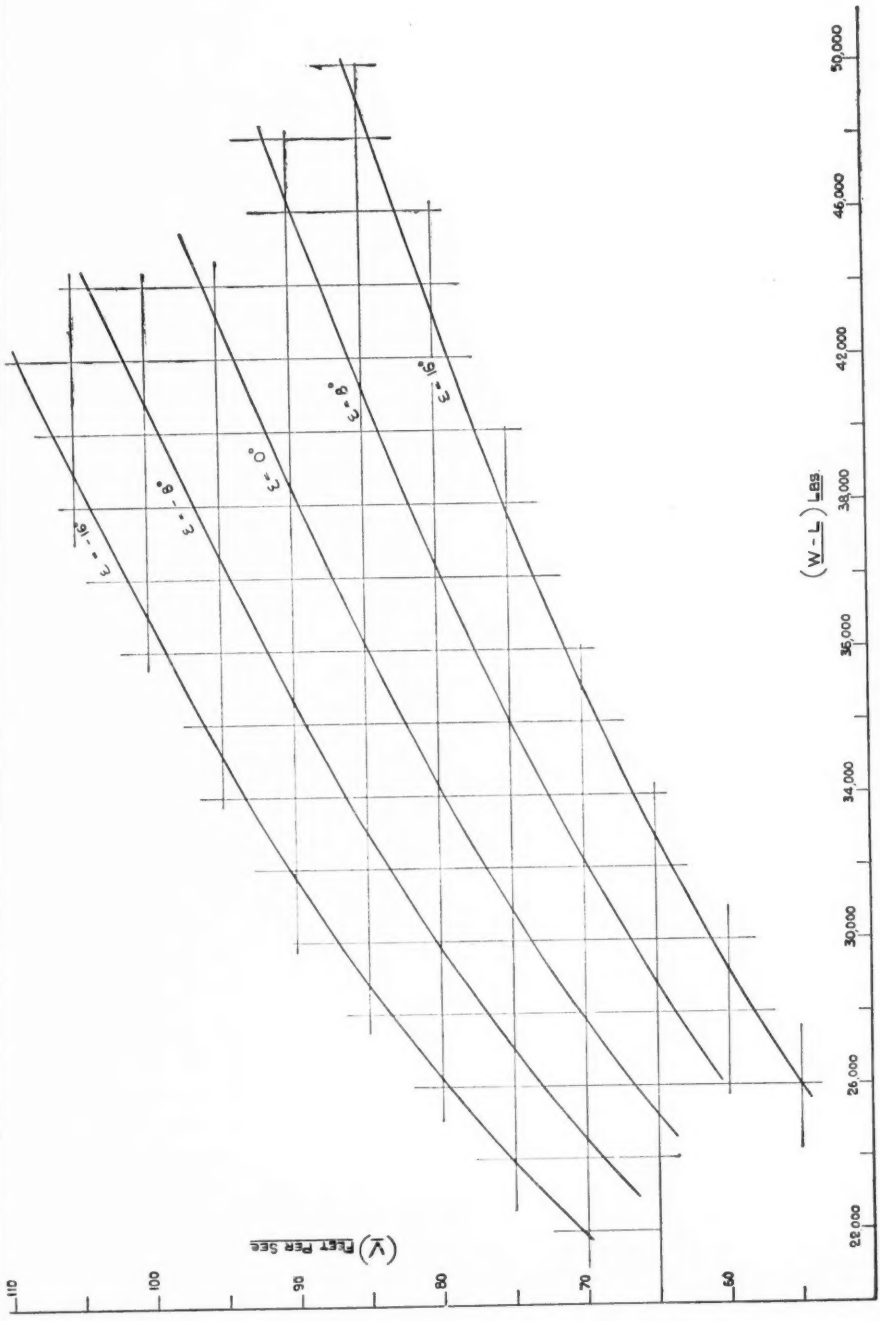

(V) Feet Per Sec.
110
100
90
80
70
60
(W-L) Lbs.
22,000
26,000
30,000
34,000
38,000
42,000
46,000
50,000
ε = 16°
ε = 8°
ε = 0°
ε = -8°
ε = -16°

FIG. 24.

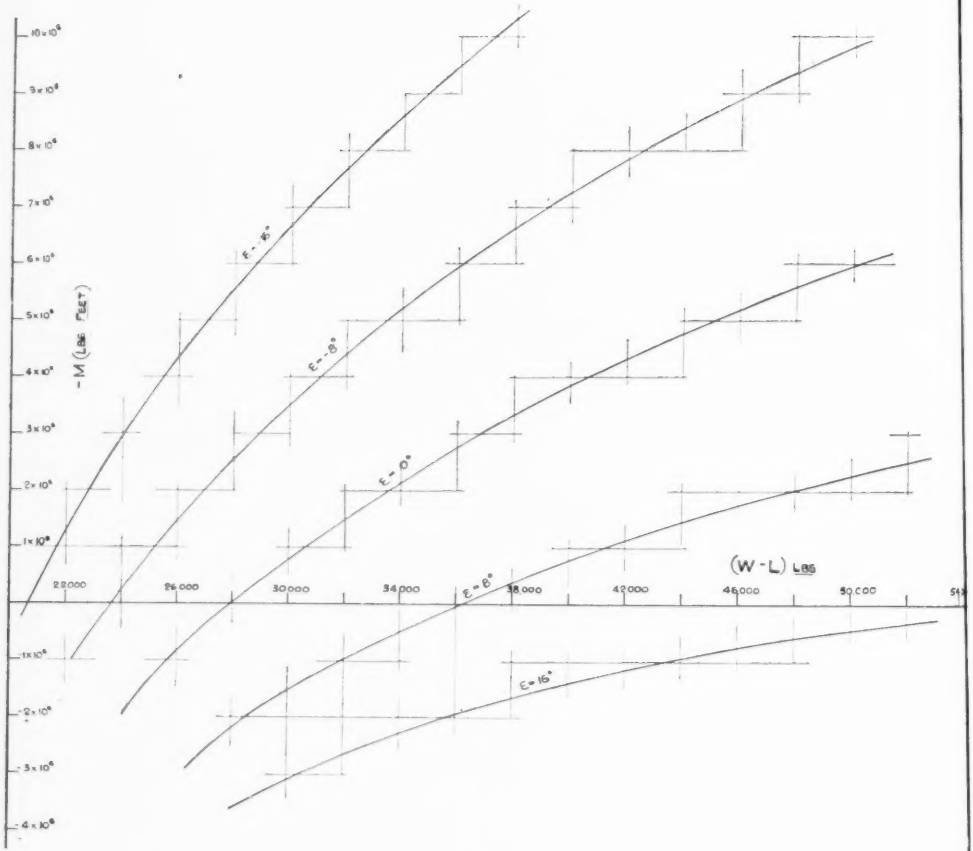

10 x 10⁶
9 x 10⁶
8 x 10⁶
7 x 10⁶
6 x 10⁶
5 x 10⁶
4 x 10⁶
3 x 10⁶
2 x 10⁶
1 x 10⁶
-M (LBS FEET)
ε = 4°
ε = 8°
ε = 0°
ε = 8°
ε = 16°
22000
26000
30000
34000
38000
42000
46000
50000
(W - L) LBS
1 x 10⁶
2 x 10⁶
3 x 10⁶
4 x 10⁶

FIG. 25.

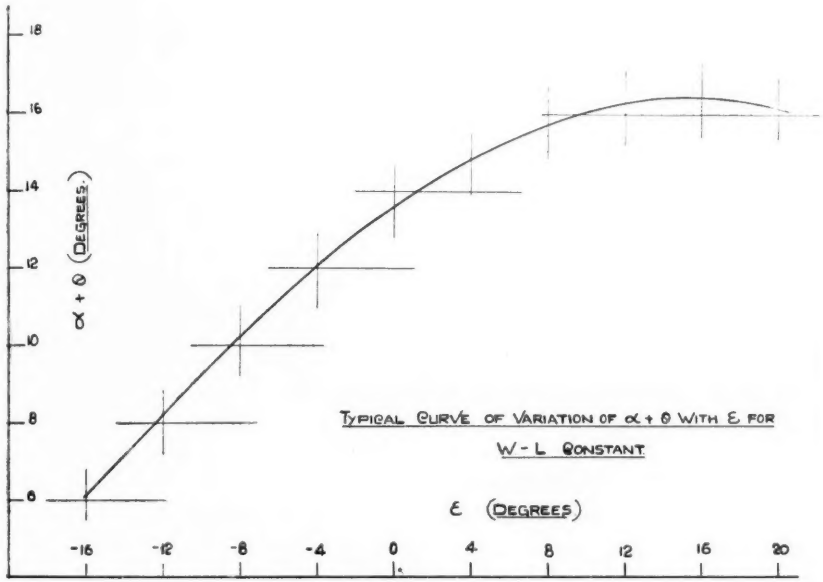

18
16
14
12
10
8
6
α + θ (DEGREES.)
TYPICAL CURVE OF VARIATION OF α + θ WITH ε FOR
W - L CONSTANT
ε (DEGREES)
-16
-12
-8
-4
0
4
8
12
16
20

FIG. 26.

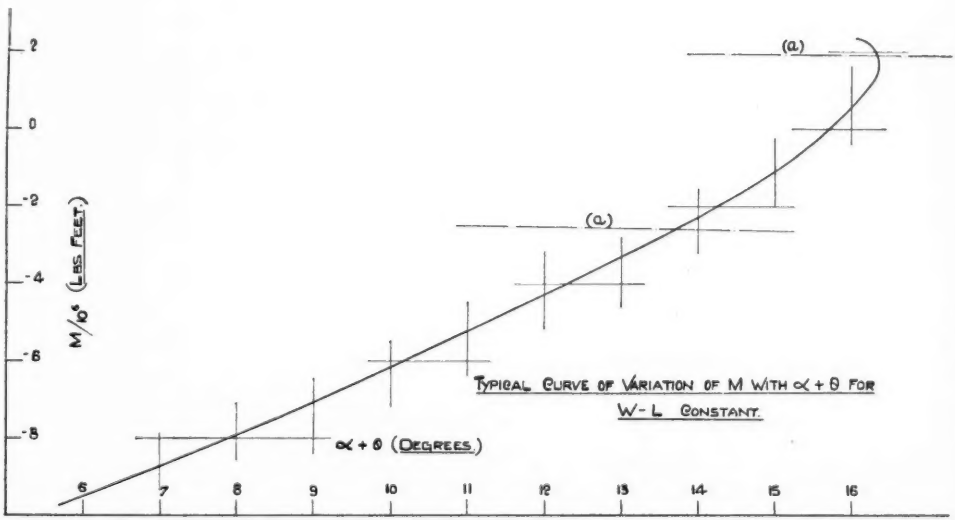

2
0
-2
-4
-6
-8
M/10⁶ (LBS. FEET.)
(a)
(a)
TYPICAL CURVE OF VARIATION OF M WITH α + θ FOR
W - L CONSTANT.
α + θ (DEGREES.)
6
7
8
9
10
11
12
13
14
15
16

FIG. 27.

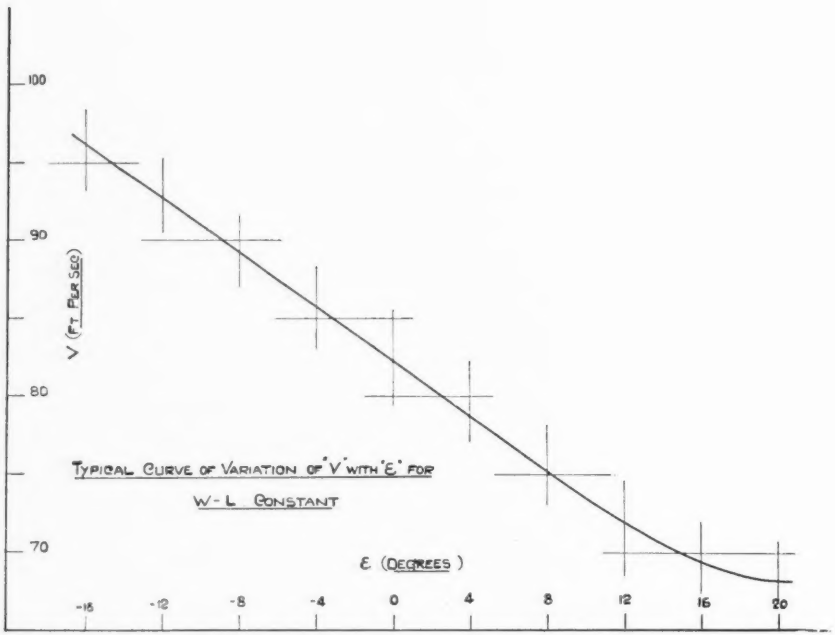


FIG. 28.

Superimposing the first of these on the (*a*) curve (Fig. 27) for this particular gasbag in the heavy condition, the $\alpha+\theta$ value for the case is obtained where the two curves cross.

From Figs. 26 and 28, the corresponding values of ϵ and V follow. The distribution of shearing forces and bending moments can then be obtained.

§9.35. The total shearing force at the tail due to the W and L system is $-(W-L)\cos\alpha+\theta$ and the total aerodynamic shear at the tail due to the A system is $(W-L)\cos\alpha+\theta=-(mZ)\,V^2$.

The shearing forces along the ship due to the aerodynamic loading of the hull are obtained by multiplying the ordinates of the curve in Fig. 8 of the appropriate θ by V^2. A typical set is shown in column 2 of Table 6*a*.

The difference between the final value and $(W-L)\cos\alpha+\theta$ gives the final value in the next column, where the shearing forces due to the aerodynamic fin loading are given. The fin values follow by proportion as explained in §7.7. In column 4 are the shearing forces due to the W and L systems, and finally, by addition, the net shearing forces are obtained in column 5.

§9.36. The total aerodynamic bending moment at the tail is equal to $BM+n\,(W-L)\cos\alpha+\theta$ where n is the distance along the axis of the tail point from the C.B. From the curves of Fig. 10 the portion of it due to the hull alone is known, and hence the B.M. at the tail due to the forces on the fins is known by subtraction (*cf.* Table 6*b*). This can be checked, as it should equal the fin aerodynamic shearing force at the tail multiplied by the distance of the fin centre of pressure from the C.B.

The bending moments corresponding with the term $Wh_W \sin \overline{\alpha+\theta}$ in equation (3), are obtained by integrating from the nose the products of the components parallel to the axis of the weights in the ship and the perpendicular distances of their C.G.'s from the axis. Referring to Table 6*b*, the final value in column 5 should agree with the value $Wh_W \sin \overline{\alpha+\theta}$.

The bending moments corresponding with the $Lh_L \sin \overline{\alpha+\theta}$ term are obtained by integrating from the nose the products of the components parallel to the axis of the gas lift forces in the ship and the perpendicular distances of their centres of buoyancy from the axis. The final value in the column 6, Table 6*b*, should agree with the value $Lh_L \sin \overline{\alpha+\theta}$.

The bending moments due to the thrust and the longitudinal aerodynamic force of the cars is obtained as indicated in §9.2. These are written in column 4.

The final bending moments, due to the algebraic addition of the components, are shown in column 8.

TABLE 6a.

SHEARING FORCES (LBS.).

1	2	3	4	5
Bay or Frame.	Hull (Aerod.)	Fins (Aerod.)	Weights and Lifts (Static)	Resultant Shear.
0—1f	-968	—	+1824	+856
1f—1a	−1993	—	2402	409
1a—2f	−3843	—	2979	−864
2f—2a	−5436	—	3060	−2376
2a—3f	−7686	—	3142	−4544
3f—3a	−9892	—	1688	−8204
3a—4f	−11671	—	237	−11434
4f—4a	−13208	—	355	−12854
4a—5f	−14378	—	473	−13905
5f—5a	−15373	—	4372	−11001
5a—6f	−16025	—	8274	−7751
6f—6a	−16568	—	12352	−4216
6a—7f	−16823	—	16420	−403
7f—7a	−16993	—	16345	−648
7a—8f	−17079	—	16284	−785
8f—8a	−17050	—	16540	−510
8a—9f	−16938	—	16785	−153
9f—9a	−16682	—	16719	38
9a—10f	−16083	—	16639	556
10f—10a	−15913	—	16593	685
10a—11f	−15088	—	16556	1469
11f—11a	−14233	—	16490	2257
11a—12f	−13236	—	16440	3204
12f—12a	−12240	−58	16365	4068
12a—13f	−11330	−317	16289	4643
13f—13a	−10447	−863	16314	5005
13a—14f	−10020	−2387	16335	3928
14f—14a	−9679	−4321	16480	2481
14a—15f	−9536	−6616	16645	763
15f—15a	−9394	−7564	17050	93
15a—16	−9280	−7736	17448	432
16—tail	−9280	−8021	17301	0

§9.37. The total aerodynamic longitudinal force is, apart from the cars, $(mX)\,V^2$. The portion due to the hull alone is deduced from curves like those of Figs. 12 and 16. The fin portion is deduced from the fin longitudinal force diagram. These are shown tabulated in columns 2 and 3 of Table 6*c*. The longitudinal forces due to the components of the weights and lifts in the axial direction are given in column 5; they can be directly obtained from column 4 of Table 6*a* by multiplying it by $\tan\overline{\alpha+\theta}$

The forces due to the difference of thrust and car drag are shown in column 4 and the net longitudinal forces in column 6. Tables 6*a*, *b*, *c*, are illustrated by the diagrams of Figs. 29*a*, *b*, *c*.

TABLE 6b.

BENDING MOMENTS (TONS FT.).

1	2	3	4	5	6	7	8
Frame Face.	Hull (Aerod.)	Fins (Aerod.)	Thrusts and Drag of Cars.	$Wbw\sin\overline{\alpha+\theta}$ (Static)	$Lbl\sin\overline{\alpha+\theta}$ (Static)	Weights and Lifts (Static)	Resultant B.M.
0	0	—	—	1	0	4	5
1f	0	—	—	1	0	15	16
1a	0	—	—	6	0	20	26
2f	−34	—	—	6	0	61	39
2a	−42	—	—	27	0	71	57
3f	−167	—	—	27	0	132	−9
3a	−205	—	—	510	0	137	−8
4f	−413	—	—	510	0	142	−262
4a	−459	—	−83	133	0	147	−262
5f	−697	—	−83	133	0	152	−490
5a	−755	—	−83	197	0	173	−469
6f	−980	—	−83	197	0	299	−568
6a	−1047	—	−83	255	0	397	−528
7f	−1293	—	−83	255	0	593	−529
7a	−1368	—	−83	313	0	664	−475
8f	−1618	—	−83	313	0	962	−427
8a	−1689	—	−83	382	0	983	−407
9f	−1960	—	−83	382	0	1754	−408
9a	−2039	—	−151	456	0	1312	−422
10f	−2310	—	−151	456	0	1606	−299
10a	−2377	—	−151	512	0	1662	−354
11f	−2652	—	151	512	0	1976	−325
11a	−2702	—	−193	571	0	2032	−288
12f	−2961	−1	−193	571	0	2371	−214
12a	−3003	−1	−193	606	0	2419	−172
13f	−326	−8	−193	606	0	2761	−70
13a	−3261	−10	−193	619	0	2801	−49
14f	−3428	−47	−193	619	0	3065	15
14a	−3445	−59	−193	621	0	3093	17
15f	−3582	−161	−193	621	0	3338	23
15a	−3595	−172	−193	621	0	3356	18
16	−3844	−414	−193	621	0	3830	0

§9.38. It will be appreciated that the labour involved in the determination of the equilibrium conditions (§9.3) would be considerably less if the value of $(W-L)$ were *always* the same wherever the deflation took place; and if reasons were found for the elimination of Cases 5-8 in Table 3, or if some simple way of taking account of them were devised, the labour is still further lightened. It is suggested that these simplifications may be introduced with little loss in accuracy, if the constant $(W-L)$ be made equal to the lift of the largest bag in the ship together with the lift that would be lost from the bags on either side of it if the uniform deflation of Cases 5-8 occur.[24] The conditions may be more

TABLE 6c.

LONGITUDINAL FORCES (LBS.).

1	2	3	4	5	6
Bay or Frame.	Hull (Aerod. press. & Skin fric.)	Fins (Skin fric.)	Thrust.	Weights and Lifts (Static)	Resultant Long Force.
0—1f	1879	—	—	524	2403
1f—1a	2167	—	—	690	2857
1a—2f	2042	—	—	850	2897
2f—2a	1677	—	—	879	2556
2a—3f	943	—	—	902	1845
3f—3a	200	—	—	485	685
3a—4f	−416	—	—	68	−351
4f—4a	−1039	—	—	102	−937
4a—5f	−1532	—	—	136	−1396
5f—5a	−1834	—	−1476	1255	−2005
5a—6f	−2053	—	−2952	2375	−2631
6f—6a	−2210	—	−2952	3545	−1617
6a—7f	−2244	—	−2952	4703	−483
7f—7a	−2221	—	−2952	4902	−481
7a—8f	−2223	—	−2952	4974	−391
8f—8a	−1951	—	−2952	4730	−168
8a—9f	−1632	—	−2952	4818	235
9f—9a	−1279	—	−2952	4799	569
9a—10f	−823	—	−2952	4771	1001
10f—10a	367	—	−4428	4764	704
10a—11f	134	—	−5904	4752	−1018
11f—11a	664	—	−5904	4733	−508
11a—12f	1162	—	−5904	4674	−69
12f—12a	1592	—	−6642	4698	−353
12a—13f	1948	—	−7380	4675	−758
13f—13a	2252	7	−7380	4682	−440
13a—14f	2378	16	−7380	4680	−298
14f—14a	2469	40	−7380	4730	−141
14a—15f	2480	90	−7380	4778	−33
15f—15a	2460	135	−7380	4894	108
15a—16	2363	165	−7380	4865	13
16-tail	2236	178	−7380	4967	0

[24] There are, of course, four different constant values for $(W-L)$ implied—one for each of the conditions (Wa, La) . . . (Wd, Ld), §§1.21, 1.22.

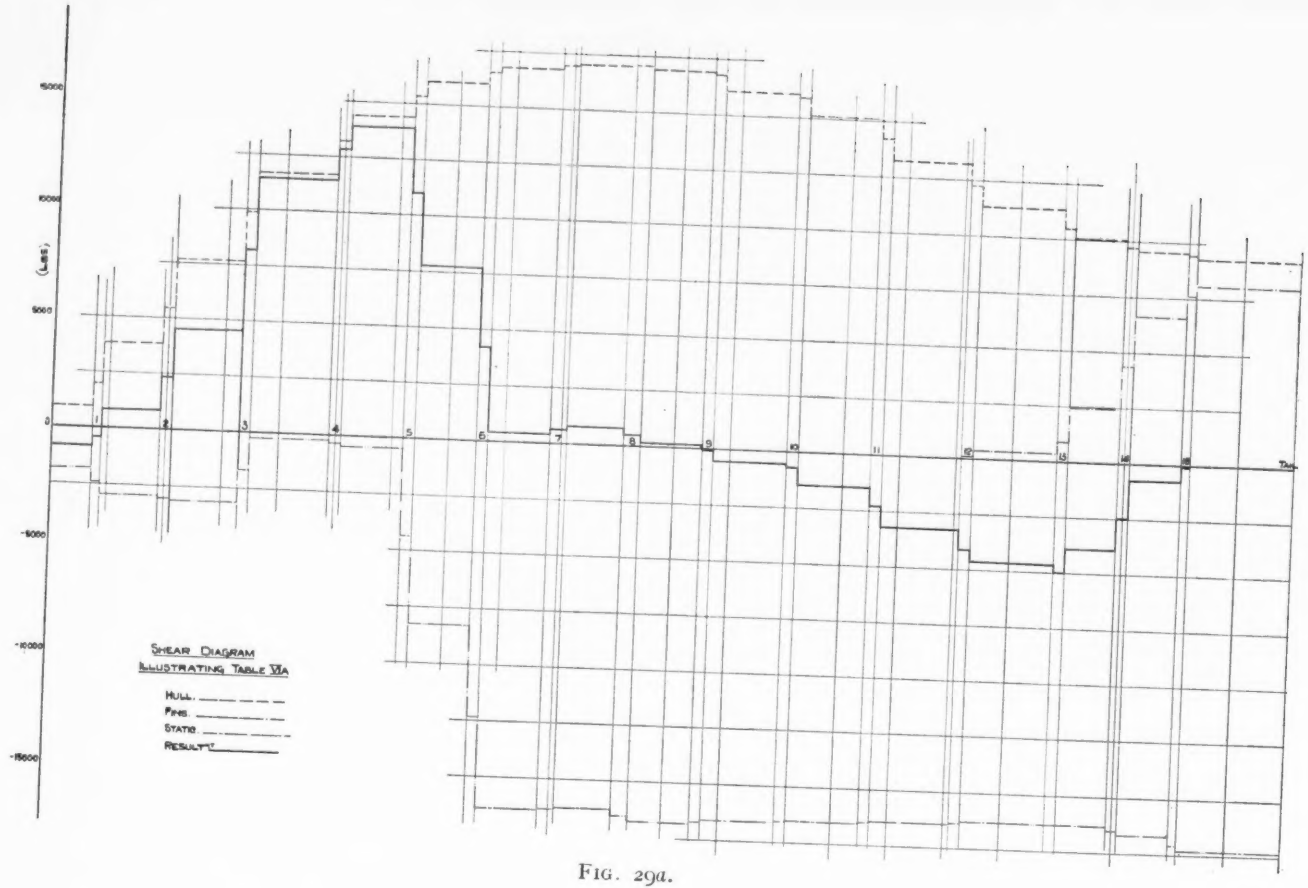

15000
10000
5000
(Lbs.)
0
-5000
-10000
-15000
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Fore
SHEAR DIAGRAM
ILLUSTRATING TABLE VIA
HULL
FINS
STATIC
RESULT

FIG. 29a.

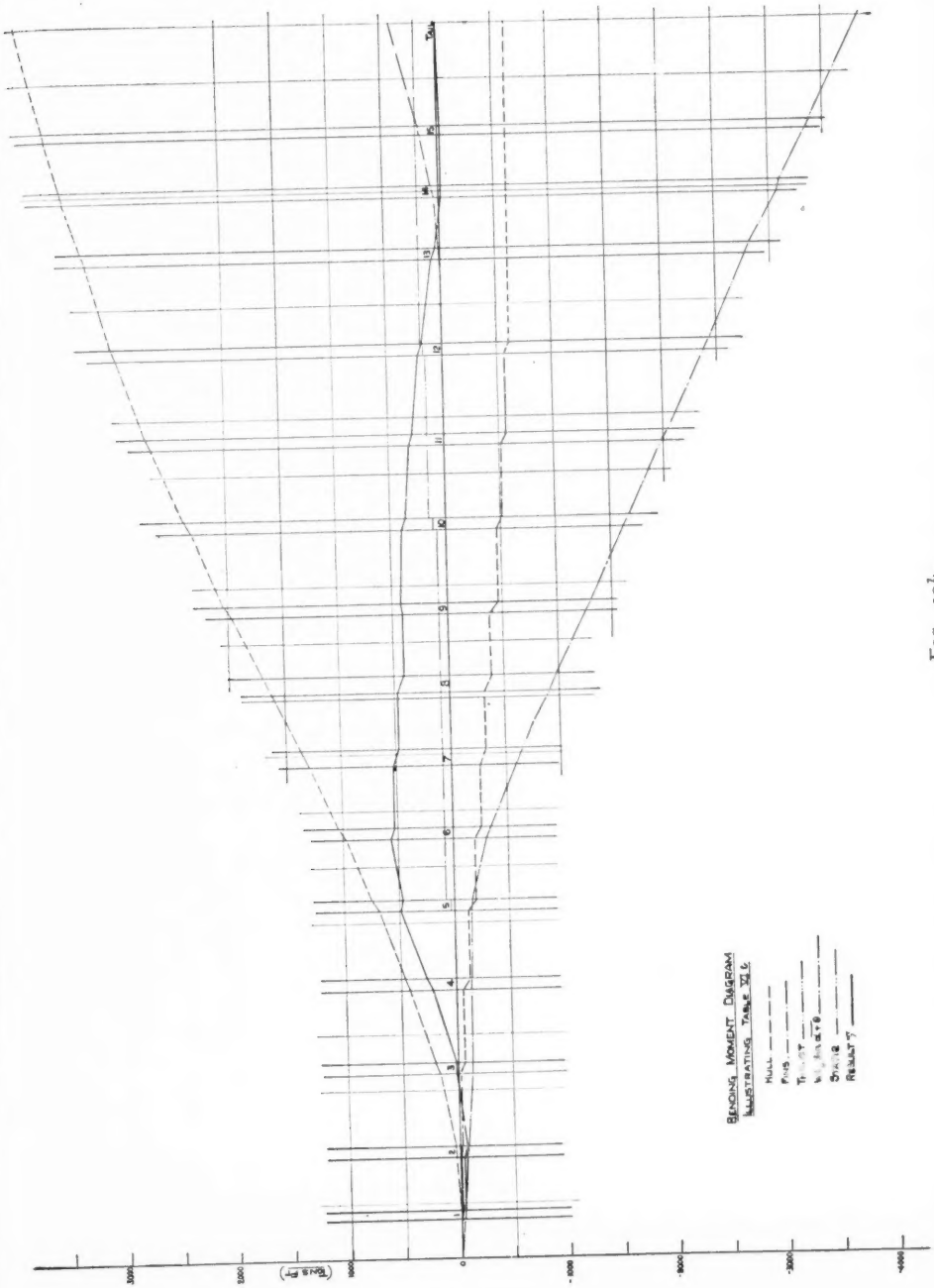


FIG. 29b.

FIG. 29a.

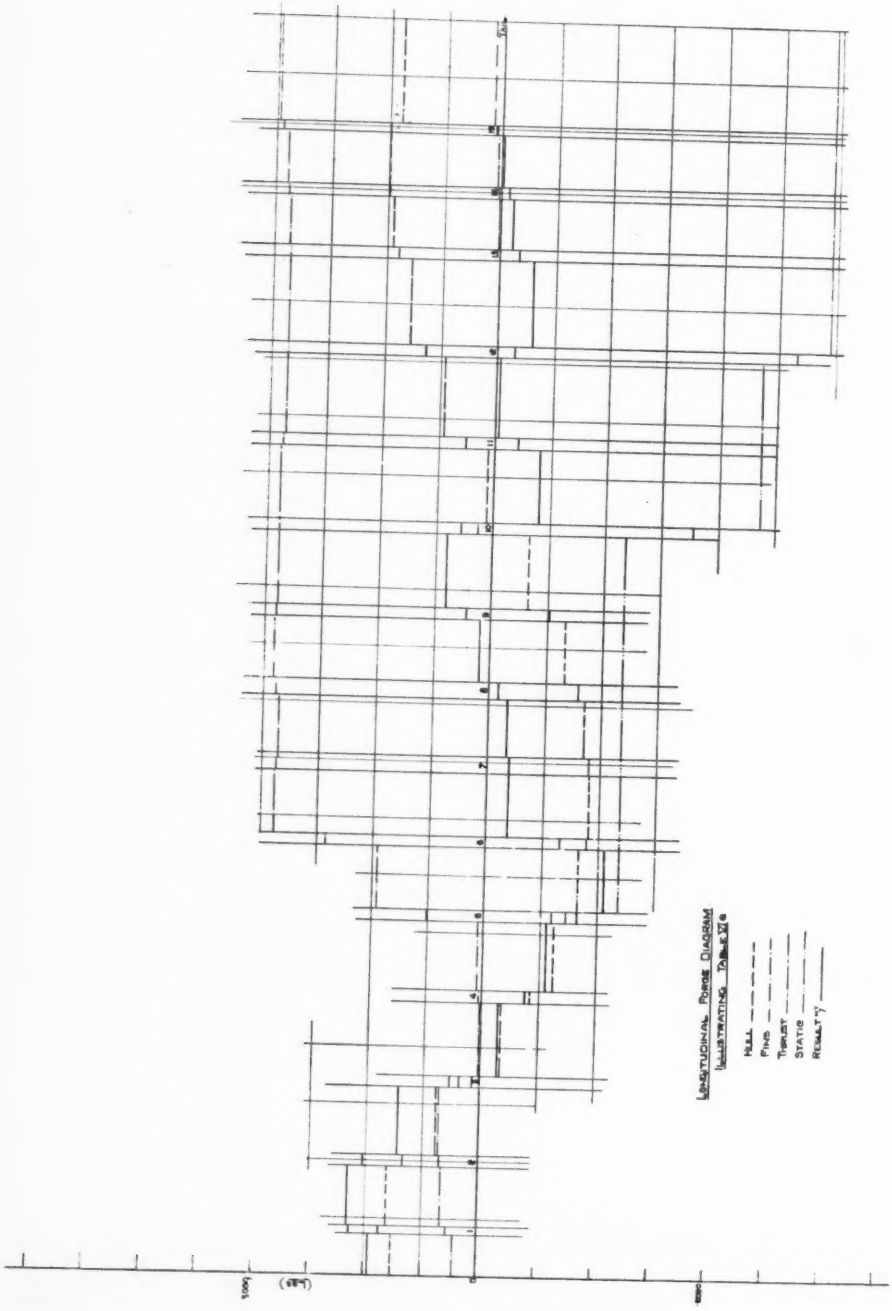

LONGITUDINAL FORCE DIAGRAM
ILLUSTRATING TABLE XI
HULL
FINS
THRUST
STATIC
REALITY
(FT)

FIG. 29c.

severe than in the more logical cases developed and listed in Table 3, but the saving in time commends itself to the designer, whose methods must, with increasing experience, tend to empiricism and who, in consequence, will always prefer a safe device to an exact theory when he is satisfied that the former will serve for design purposes with very little depreciation of design economy. In the present instance, however, the method which more closely represents actual conditions can scarcely be termed an "exact theory," and before it is discarded in favour of the "safe device" a considerable number of comparisons of results on the same ship using the two methods are necessary.

The necessary curves for use in the case of $(W-L)$ constant can be obtained by cross plotting the curves of the general solution at the chosen $(W-L)$ value. They may be obtained more directly, however. First of all, equations (1) and (2) are solved simultaneously for the two unknowns α and V. This is most conveniently done by writing them

$$\sin \alpha + \theta = \{ (mX + mX_q) V^2 + K - kV \} / (W - L) \quad . \quad . \quad (1a)$$

$$\cos \alpha + \theta = - mZ_q V^2 / (W - L) \quad . \quad . \quad . \quad . \quad (2a)$$

and finding the value of V which in (1a) and (2a) gives the same $\alpha + \theta$ value. With practice, it is not found always necessary to resort to plotting to do this. The value of M then follows from equation (3) and the subsequent procedure is as has already been explained in §§9.34, 9.35, 9.36, 9.37.

§9.4. *Cases 45-52*

In dealing with the superheating cases, the same equations of equilibrium apply, though care is necessary in assigning the signs of the quantities. Details of the procedure necessary to obtain the primary actions are similar to those given already for positive pitch cases in §9.38.

The $(W-L)$ values which are, apart from gasbag deflations, negative, are calculated in the way indicated in §6.4.

FIG. 29c.

§10. Details of Component A for Circling Flight at 0° Pitch

§10.1. *Experimental Data Required*

Measurements on the model scale of the forces occurring in circling flight cannot be made without the help of the whirling arm. In the absence of such tests, experiments made in the wind tunnel are used to provide tables of longitudinal force mX_β, lateral force mY_β, and yawing moment CN_β for

(*a*) The bare hull over a range of angles of yaw of say $-20°$ to $20°$.

(*b*) The hull with the tail surfaces in position, elevators amidships, with rudders at different angles—say $0°$, $\pm 5°$, $\pm 10°$, $\pm 20°$, $\pm 30°$—over the same range of yaw angles at each rudder angle.

(*c*) The complete ship with flaps amidships, over the same range of rudder angles.

In the case of ships whose hulls and fins are in front elevation symmetrical in the same way about horizontal and vertical axes, tables (*a*) and (*b*) can be deduced straightway from tables (*a*) and (*b*) of §7.1, but the asymmetry of the cars necessitates as a rule the tests for table (*c*).

In addition, the rate of change of the yawing moment with angular velocity (CN_r) for the same range of yaw angles, is required. This can be obtained in the wind tunnel by an oscillation experiment.[25] The rates of change, mY_r and mX_r respectively, of lateral force and longitudinal force cannot however be determined without the use of the whirling arm or full-scale experiment; the

[25] Jones, *loc. cit.* Journal of the Royal Aeronautical Society, Vol. XXVIII., p. 107.

former is required in the ensuing work, and the value used is an approximation based on the evidence of full-scale experiment.

§10.2. *Equations of Equilibrium*

In circling flight with no pitch the equations of equilibrium are, referring to Fig. 30

$$mX + r_o mX + T - (mV^2/R)\sin\beta = 0 \quad . \quad . \quad . \quad . \quad (1)$$
$$mY + r_o mY_r - (mV^2/R)\cos\beta = 0 \quad . \quad . \quad . \quad . \quad (2)$$
$$CN + r_o CN_r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where β = the angle of yaw measurement at the C.B.
m = the mass of the ship in slugs.
mX = the longitudinal force at rectilinear yaw β in lbs.
mY = the lateral force at rectilinear yaw β in lbs.
CN = the yawing moment at rectilinear yaw β in lbs.
r_o = angular velocity = V/R.
$mX_r = (\partial/\partial r)\, mX$.
$mY_r = (\partial/\partial r)\, mY$.
$CN_r = (\partial/\partial r)\, CN$.
R = radius of turning circle.

On account of the absence of information on the magnitude of X_r the first equation cannot be solved to find V. The terms in equations (2) and (3) are all functions of V^2, so the speed in circling flight must be assumed at some likely value. The Airworthiness of Airships Panel recommend[26] a speed equal to 75 per cent. of the estimated maximum speed, and this should be adopted until a means of predicting the true speed is discovered.

In addition to equations (2) and (3), it has been shown[27] that

$$R\beta = K\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where λ is the distance between the C.B. and the centre of pressure of the forces on the fins and rudders, and K is a constant. This constant may be obtained on certain assumptions[28] from equations (2) and (3) as

$$-m\beta\,[V\{Y_r)_n - V\cos\beta\} + V(CN_r)_n/m\lambda]/(CN_\beta + \lambda mY_\beta)$$

and may vary from roughly .9 to 1.1 for a range of fineness ratios of about 9 : 1 to 4.5 : 1. For R.101, which has a fineness ratio of 5.5 : 1, K is approximately 1.0.

Equations (2) and (3) may be written

$$mY + (V/R)\, mY_r = (mV^2/R)\cos\beta \quad . \quad . \quad . \quad (2a)$$
$$CN + (V/R)\, CN_r = 0 \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

Then from (3a)

$$(1/R) = -(CN/VCN_r) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From (4)

$$(1/R) = (\beta/K\lambda) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The expression on the right hand side of (6) which is independent of V may be evaluated from the experimental data and plotted for a given rudder angle against β.

On the same diagram $\beta/K\lambda$ is plotted against β and the intersection of the two curves gives the radius of the turning circle and the angle of yaw at the C.B. for the rudder angle chosen.

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[26] R. & M. 970, p. 9.

[27] "The Aerodynamical Characteristics of the Airship as deduced from Experiments on Models," by R. Jones, Journal of the Royal Aeronautical Society, Vol. XXVIII., p. 112.

[28] λ is best found from the tables for mY_β for hull only and for hull and fins with rudders amidships, taking the average value for the first 8° of yaw.

Now consider equation (2*a*). It is convenient to work at $V=1$ f.p.s., and to obtain the force distributions at V f.p.s. by multiplying by V^2, so the equation may be written

$$(mY)\,V^2+(V/R)\,V\,(mY_r)=(m/R)\,(\cos\beta)\,V^2$$

where terms in brackets represent forces at 1 f.p.s.

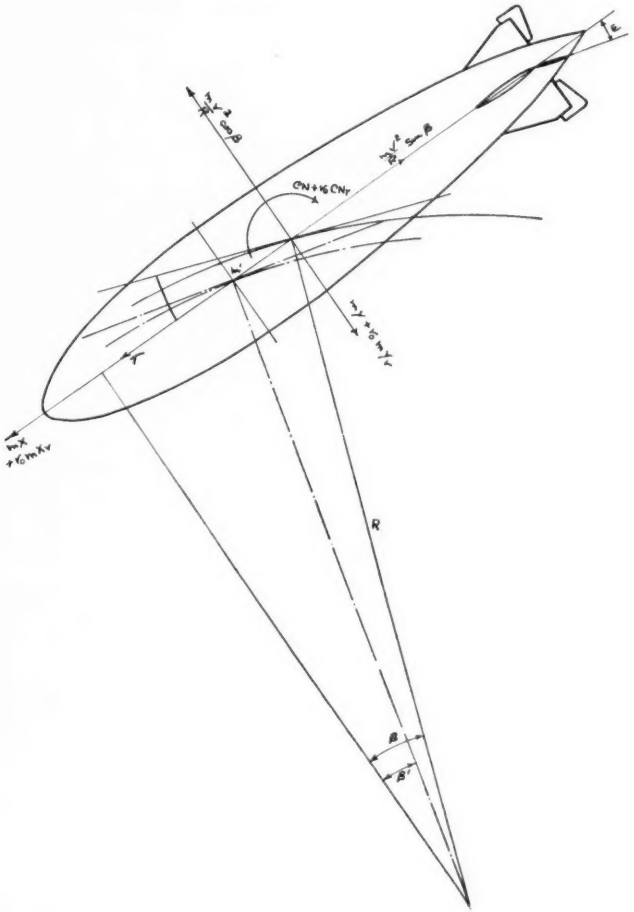


FIG. 3c.

Therefore, *at* 1 f.p.s.

$$(mY)+(1/R)\,(mY_r)=(m/R)\cos\beta \quad . \quad . \quad . \quad (2b)$$

or, dividing the terms to represent hull and fin forces,

$$(mY)_h+(mY)_f+(mY_r/R)_h+(mY_z/R)_f=(m/R)\cos\beta \quad . \quad . \quad (2c)$$

Now $(mY)_h$ $(mY)_f$ and $m/R\cos\beta$ are all directly obtainable. The first two can be found from the tables of forces on the model for rectilinear yaw, and the third from the knowledge of the ship's displacement. Then, if it be assumed

that $(Y_r)_h = .1$, which is a value indicated by experiment, all the terms in the equation become known, whence

$$[mY]_h = (mY)_h + (mY_r/R)_h \quad . \quad . \quad . \quad . \quad (8)$$

$$[mY_g]_f = (mY)_f + (mY_r/R)_f \quad . \quad . \quad . \quad . \quad (9)$$

where the square brackets refer to *total* forces on hull and fins respectively.

Dealing in a similar way with equation (3*a*), it may be written

$$(CN)_h + (CN)_f + (1/R)(CN_r)_h + (1/R)(CN_r)_f = 0 \quad . \quad . \quad (3c)$$

The first three terms are known from the experimental tables, whence the last follows; so

$$[CN]_h = (CN)_h + (1/R)(CN_r)_h = D$$

say and

$$[CN]_f = (CN)_f + (1/R)(CN_r)_f = -D$$

where the square brackets refer to *total* moments on hull and fins respectively.

§10.3. *Lateral Force Distribution*

The diagram of lateral force distribution may now be constructed, and recourse is had to hydrodynamical work on the spheroid used in §7.2. When the yaw at the origin of the spheroid is β' the lateral force distribution on it is given by

$$P = \{\tfrac{1}{2}\rho V^2 (2\pi b^2/a)\cos\beta'\}\sin^2\psi\,[(1+L)(1+M)\cos\psi\sin\beta' - (1+L)(1+e^2N\cos 2\psi)(a/R) + (\sin^2\psi + (b^2/a^2)\cos^2\psi)(a/R)]\,[\sin^2\psi + (b^2/a^2)\cos^2\psi] \quad . \quad . \quad (1)$$

where

$$N = -[(3/2e^3)\log(1+e)/(1-e) - 3 - e^2/(1-e^2)]/[(3/2)\{(2-e^2)/e^3\}\log(1+e)/(1-e) - 6 + e^2/(1-e^2)]$$

and the other terms are as defined in §7.2.

This expression is directly applicable to the spheroid at the nose of the airship when the yaw at the origin of this spheroid is taken. In the majority of airship shapes which have approximately spheroidal noses this origin occurs at the first maximum diameter of the ship, so β' must be the angle of yaw at this position and not the angle of yaw at the C.B. (see Fig. 30).

Using equation (1), P is plotted for $V = 1$ f.p.s. for the spheroidal portion of the ship's shape, and the diagram is completed by using the values of $[mY]$ and $[CN]$ found in §10.2 and the methods of §7.2.

If desired, a set of diagrams of lateral force distribution in circling flight for different values of β may be constructed to form a "family of curves," in the manner of Fig. 7, but if the proposal to consider only two different rudder angles is adhered to, this will not be necessary. A typical diagram of lateral force distribution is given in Fig. 31.[29]

The diagrams of shearing force and bending moment due to the hull forces are obtained in the ways given in §7.3 and §7.4. With regard to the fins, the diagrams of Figs. 17, 18, 19, and the methods of §7.7 may be used.

§10.4. *Curves of Pressure Distribution*

In the case of pitched flight the building up of curves of pressure distribution in the way described in §7.5 is not generally required if pressure plotting on the model in the wind tunnel has been done. Without the whirling arm experiments however, which are more laborious to perform than wind tunnel experiments, the pressure distribution in circling flight cannot be obtained so the methods of §7.5 have to be used. The labour involved is still greater than that of §7.5, however, as the expression for pressure at a point for a spheroid in circling motion is[30]

[29] The local aerodynamic lateral forces due to the cars are not specially represented.

[30] *loc. cit.* §7.2, Trans. Royal Soc., p. 235.

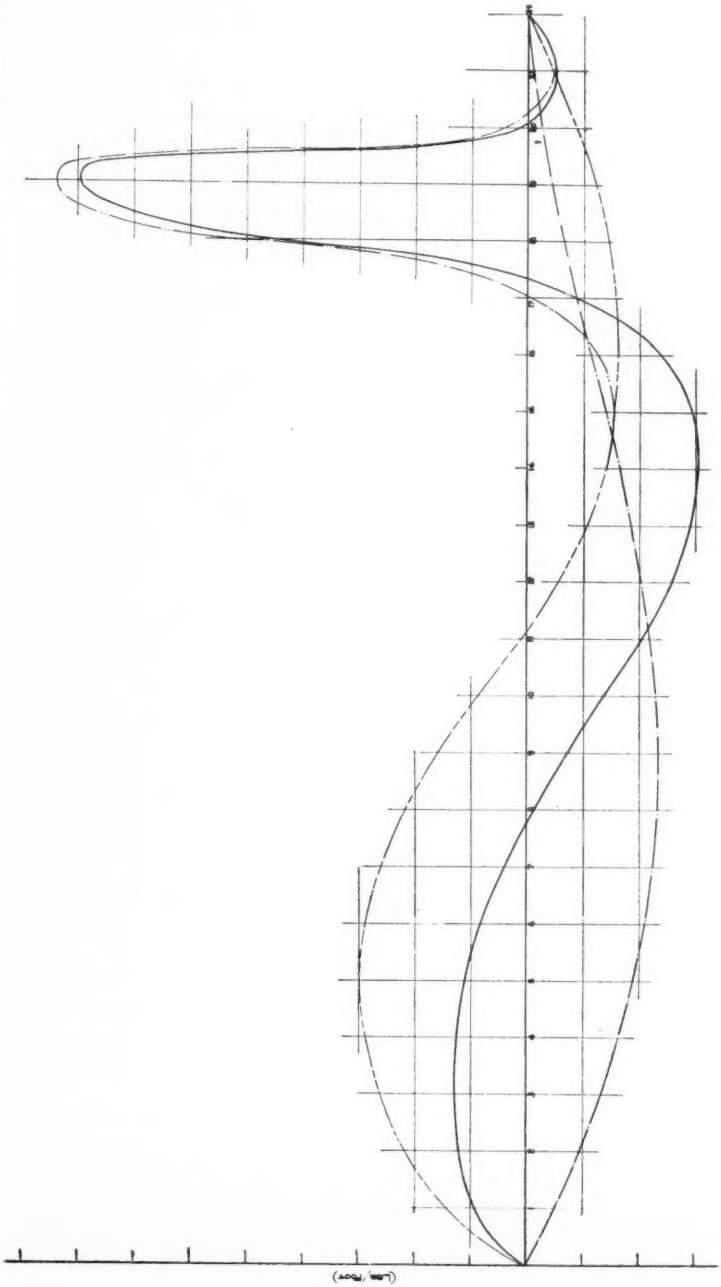


FIG. 31.

Distribution of Lateral Force.
Ship Turning at 1 foot per sec.

Aerodynamic Force Distribution due to Hull — - - - - - -
Due to Fins — - - - - -
Centrifugal Force Distribution — - - - -
Resultant Distribution ———

$$p = a_0 + a_1 \cos w + a_2 \cos^2 w + a_3 \sin^2 w + (b_1 \cos w + b_2 \cos^2 w + b_3 \sin^2 w)\,\Omega + (c_2 \cos^2 w + c_3 \sin^2 w)\,\Omega^2$$

where $a_0 \ldots a_3$, $b_1 \ldots b_3$, c_2, c_3, are as given in Appendix IV., where their values for R.33 are given.

If time cannot be spared for this work when designing, there is little doubt that the distributed loads on the girders due to the true pressure distribution may be safely replaced by those for a pitched flight case at somewhat greater angle and speed.

§10.5. *Longitudinal Loads*

If the appropriate pressure distribution has been found, the longitudinal loads may be summed as in §7.6; otherwise, those from a suitable pitch case may be used.

§11. Cases 53-68 (Cases of Circling Flight)

§11.1. *Forces due to Component W*

The systems due to weight and lift which are to be combined with the aerodynamic forces which have just been discussed to form Cases 53-68 are themselves in equilibrium in the vertical plane. Due to the circling motion of the ship, however, centrifugal forces arise which (see equation (2) of §10.2) balance the lateral aerodynamic forces and contribute (see equation (1) of §10.2) to the longitudinal forces.

Consider any mass m' in the ship distant x from the nose, shown in Fig. 30 at the maximum diameter position to avoid confusion in the diagram. Then the angle of yaw at the position of the mass is β_m, where

$$\tan \beta_m = (R \sin \beta - c + x)/R \cos \beta$$

its radius of turn is

$$R\,(\cos \beta / \cos \beta_m)$$

and its centrifugal force is

$$Q = m'\,(R \cos \beta / \cos \beta_m)\,\Omega^2$$

where

$$\Omega = V/R.$$

The lateral component of this force is

$$Q \cos \beta_m = m' R \cos \beta \Omega^2 = m'\,(V^2/R) \cos \beta$$

and the longitudinal component is

$$Q \sin \beta_m = m' R \cos \beta \Omega^2 \tan \beta_m = m'\,(V^2/R) \cos \beta \tan \beta_m.$$

Centrifugal forces arising from the mass of the girders and the fabric attached to them provide another source of distributed loading on the girders.

In building up tables of primary actions for a case of turning, the vertical shearing forces and the corresponding bending moments may be written down directly from the appropriate case among Cases 1-16.

To obtain the centrifugal force components, the masses in the ship, including the gas volumes and the air contained in the ship, are grouped at the frame faces. The lateral components are then tabulated by multiplying these masses by $(V^2/R) \cos \beta$. Fig. 31 shows a diagrammatic distribution of these forces, together with the aerodynamic lateral forces, and the resultant lateral loading. Then the appropriate values of β_m having been found, the longitudinal components are obtained by multiplying the values in the $(m'V^2/R) \cos \beta$ column by the corresponding values in a $\tan \beta_m$ column.

§11.2. *Shearing Forces, Bending Moments and Longitudinal Forces*

By tabular integration, as explained in §6.1, columns of shearing forces and bending moments due to the lateral components of centrifugal force can be found. Both columns will balance out to zero at the tail, and when they are added algebraically to the aerodynamic shearing force and bending moment figures due to hull and fins, the resultant actions for the horizontal plane result.

The longitudinal components of centrifugal forces are integrated to give the total longitudinal forces up to frame faces. These are combined as in §9.37 with the aerodynamic longitudinal forces and the thrust to give resultant longitudinal forces along the ship.

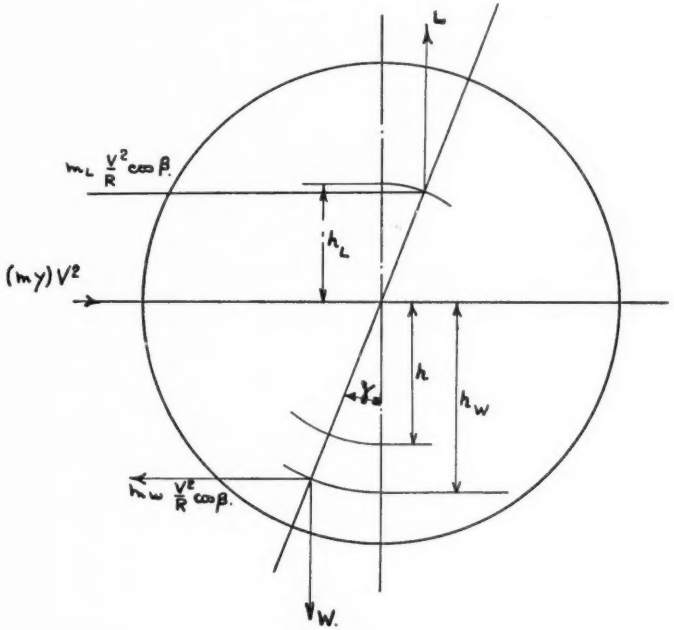


FIG. 32.

§11.3. *Rolling and Pitching Moments in Circling Flight*

Actually, as the centre of mass of the ship is below the C.B. there will be two disturbing moments acting on the ship in circling flight. If the vertical distance between the C.B. (at which the resultant aerodynamic force is supposed to act) and the centre of mass is h, then there will exist a rolling moment $(mV^2/R)\cos\beta \,.\, h$ and a moment $(mV^2/R)\sin\beta \,.\, h$, tending to pitch the ship negatively. The work described in the preceding paragraphs is really only applicable if these moments are neutralised by movement of ballast in the ship.

No attempt is made to keep an absolutely even keel in turning in actual flight, and the roll can be an important factor in the design of certain members. For example, certain members of the power car suspension system may be more heavily loaded when the ship is rolled than when it is not. Consider Fig. 32.

The rolling moment produced by the lateral and centrifugal forces, if the angle of roll is γ is

$$[mY]\ V^2h\cos\gamma=(mV^2/R)\cos\beta\,.\,h\cos\gamma=(m_wh_w-m_Lh_L)(V^2/R)\cos\beta\cos\gamma \quad (1)$$

In the equilibrium position this is balanced by the static righting moment; so

$$(m_wh_w-m_Lh_L)(V^2/R)\cos\beta\cos\gamma=(Wh_w+Lh_L)\sin\gamma \quad . \quad . \quad (2)$$

$$\therefore\ \tan\gamma=(V^2/R)\cos\beta\,(m_wh_w-m_Lh_L)/(Wh_w+Lh_L)$$

$$=(V^2/Rg)\cos\beta\,(Wh_w-Lh_L)/(Wh_w+Lh_L) \quad . \quad . \quad . \quad (3)$$

If, instead of considering the whole weight and total lift of the ship, the weight and lift concentrated at each frame are considered in turn in the same way, the angle will not prove constant along the ship. This means that a varying torque is passing along the ship. If the terms in the brackets in equation (2) be considered to apply to a particular frame, and the others to the ship as a whole, the contribution to the torque at the frame is given by

$$t=(Wh_w+Lh_L)\sin\gamma-(Wh_w-Lh_L)(V^2/Rg)\cos\beta\cos\gamma$$

and the torque across any bay of the ship is given by the sum of such elements occurring toward or aft of the bay.

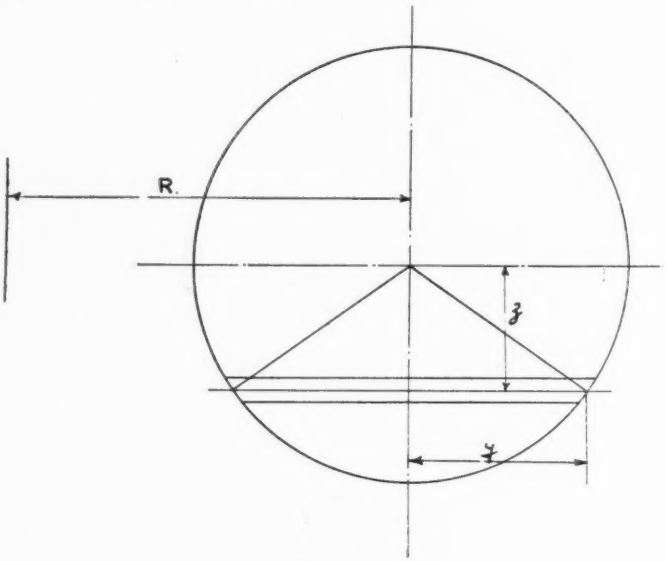


FIG. 33.

It may be assumed that the aerodynamic forces are unaffected by the roll, and the tendency to pitch may be statically corrected before the primary actions are found.

§11.4. *Centrifugal Force of Air contained in Ship*

Besides the distributed loads on the girders that have been mentioned, there is a small effect arising in circling flight which may be taken into account. Suppose the gasbags are partially deflated; there is then a large bulk of air in the ship, the centrifugal force due to which will modify the pressure on the outer cover already arrived at. Considering an elementary tube of air, as sketched in Fig. 33, it follows that at a depth z below the axis, where the beam

is $2y$ there is a change in pressure at the fabric walls at each end of $(\rho V^2/R)\,y$. There is a similar but still less important effect occurring in the gas, but the secondary load on the girders is only affected if the gasbags are allowed to bear on them.

§12. Cases 69-88 (Cases for the Ship at the Mast)

§12.1. *Pitch at the Mast*

If R_{MX} is the reaction of the masthead in the direction of the ship's axis, R_{mZ} is its reaction in the vertical plane normal to the ship's axis and c is the distance of the C.B. from the masthead, then with the notation of §9.1

$$R_{MX} + (mX + mX_c)\,V^2 - (W - L)\sin\theta = 0 \quad . \quad . \quad . \quad (1)$$

$$R_{MZ} + (mZ)\,V^2 + (W - L)\cos\theta = 0 \quad . \quad . \quad . \quad . \quad (2)$$

$$(BM - cmZ)\,V^2 + (mX_c)\,V^2\tau + W\,(c\cos\theta - h_w\sin\theta) - L\,(c\cos\theta + h_L\sin\theta) + M = 0 \quad . \quad . \quad . \quad . \quad (3)$$

These are the most general equations and are simplified in the cases to be considered where $W - L$ and M are both zero. Then

$$R_{MX} + (mX + mX_c)\,V^2 = 0 \quad . \quad . \quad . \quad . \quad (1a)$$

$$R_{MZ} + (mZ)\,V^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

$$(BM - cmZ)\,V^2 + (mX_c)\,V^2\tau - (Wh_w + Lh_L)\sin\theta = 0 \quad . \quad . \quad (3a)$$

It must be remembered, however, when shearing forces and bending moments are being built up, that all the terms in (1), (2) and (3) will be represented as $(W - L)$ is not *locally* everywhere zero. The equations are equally applicable to $+^{ve}$ or $-^{ve}$ pitch if θ is given the correct sign.

Equations (1*a*), (2*a*), (3*a*), are easily solved, as the character of the Cases 69-88 gives a pre-knowledge of V and θ.

First of all it is necessary to plot a curve of $(BM - cmZ)$ for the degree of pitch, θ, chosen. Then as values of $\sin\theta$, $mX_c\tau$ and $(Wh_w + Lh_L)$ may be substituted in equation (3*a*), the necessary $(BM - cmZ)$ follows. From the curve drawn, the elevator angle required for equilibrium is found. From the experimental data, it is then easy to find the correct values of mX and mZ, and R_{MX} and R_{mZ} then follow from (1*a*) and (2*a*).

The shear diagram for the aerodynamic forces on the hull follows at once from Fig. 8 and is tabulated. The remainder of $(mZ)\,V^2$ is due to the fins, and the table due to them is obtained by proportion from Fig. 18. The static shears are those of the particular case from 1-16, which is being dealt with, multiplied by $\cos\theta$. A fourth column is needed. This represents the constant shear due to R_{mz}. The algebraic addition of the four columns gives the net distribution of shearing forces.

The bending moment at the tail due to the aerodynamic forces is

$$\begin{aligned}&(BM - cmZ)\,V^2 + mZV^2 s\\ &= (BM)\,V^2 + mZV^2 n\\ &= (BM)\,V^2 + R_{mZ}n\end{aligned}$$

where n is as in §9.36. The bending moment at the tail due to the hull alone is known from Fig. 10, and so both hull and fin bending moment columns may be written down. The static bending moments, which are represented in equation (3) by $(W - L)\,c\cos\theta$ are obtained by multiplying those of the static case which is being incorporated by $\cos\theta$. The moments represented by $Wh_w\sin\theta$, $Lh_L\sin\theta$, $mX_cV^2\tau$, are added just as indicated in §9.36, though the last is due to car drag only, and not to thrust as well, as in §9.36.

Aerodynamic longitudinal forces are obtained as described in §9.37. The longitudinal forces corresponding with $(W - L)\sin\theta$ are found by multiplying the static shearing forces by $\tan\theta$, and the total longitudinal forces are found

by addition of these components to the constant longitudinal force represented by R_{mX}.

§12.2. *Yaw at the Mast*

Referring to Fig. 34, it is seen that the equations representing a ship in static trim, yawed at the mast and in equilibrium are

$$R_{mX} + (mX + mX_c)\, V^2 = 0 \quad . \quad . \quad . \quad . \quad (1)$$
$$R_{mY} + (mY)\, V^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$(CN)\, V^2 - c\,(mY)\, V^2 = 0 \quad . \quad . \quad . \quad . \quad (3)$$

A curve of $CN - c\,(mY)$ against β is first plotted from the experimental data for the rudder angle decided upon and the value of β which solves equation (3) is read from it. Equations (1) and (2) can then be solved for the speed adopted. Tables of shearing force, bending moment and longitudinal forces are then built up by means similar to those described in §12.1, using Figs. 7, 8, 10, if the ship has the symmetry described in §10.1.

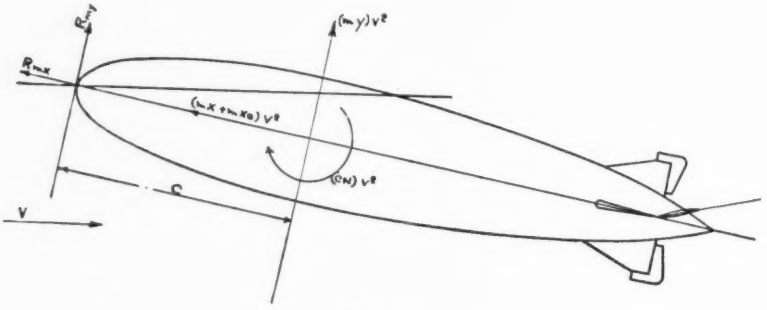


FIG. 34.

§12.3. *Pitch and Yaw at the Mast*

When a ship is both pitched and yawed at the mast, the system of aerodynamic forces it experiences in a given wind is different from that obtained by superimposing the systems for the pitch and the yaw taken separately at the same wind speed.

Consider the orientation of the ship with reference to the axes shown in Fig. 35. It is supposed that the wind is blowing in the direction OX, and that the ship is initially lying along this axis, being represented by the length OP.

Now suppose OP be rotated about O in the plane $XO\dot{Y}$ through the prescribed angle of yaw β, into the position OP_1, and then rotated about O in the plane P_1OZ, through the prescribed angle of pitch θ into the position OP_2.[31] Then the angle between the ship's axis and the wind direction is P_2OP. If this angle be called α

$$\sin^2 \alpha/2 = \sin^2 \beta/2 + \sin^2 \theta/2$$

or

$$1 + \cos \alpha = \cos \beta + \cos \theta$$

The resultant aerodynamic forces occurring will be due to the net angle between the ship's axis and the wind, and these forces will lie in the plane OPP_2. It is necessary to find the relation of this plane to the planes which, when the ship is horizontal, can be described as the vertical and horizontal planes con-

[31] Alternatively, of course, the pitch rotation may be performed first and the yaw rotation second.

taining the ship's axis. To do this, consider rectangular axes x, y, z, fixed in the ship with origin at P. Then the trace in the plane zP_2y of the plane OPP_2 is pP_2, and to find the component force distributions in the planes OP_2z and OP_2y, the values of $\cos p_1P_2p$ and $\cos pP_2p_3$ are required; as

$$P_2p_1 = OP \sin\theta, \quad P_2p_3 = OP \sin\beta,$$

$$\cos p_1P_2p = \sin\theta/(\sin^2\theta + \sin^2\beta)^{\frac{1}{2}}, \quad \cos pP_2p_3 = \sin\beta/(\sin^2\theta + \sin^2\beta)^{\frac{1}{2}}.$$

First of all, the distribution of resultant aerodynamic loading is deduced for the angle α from the curves of Fig. 7. The values so obtained are then resolved by means of the two cosines just obtained into components in the normal and lateral, or "vertical" and "horizontal" planes. Then the moment about the masthead of each of these component distributions is found by the methods of Appendix III.

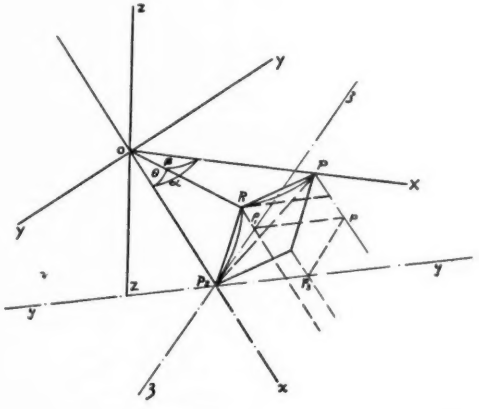


FIG. 35.

Now if the moment due to the normal hull component be written $(BM\alpha)_h \cos p_1P_2p \cdot V^2$, and if the drag of the cars is assumed to be that for an angle of inclination to the wind of α, by analogy with equation (3a) of § 12.1

$$\{(BM\alpha)_h \cos p_1P_2p + (BM\alpha)_t \cos\eta\}\, V^2 + (mX_c)_a V^2\tau - (Wh_w + Lh_L)\sin\theta = 0 \quad (1)$$

where $(BM\alpha)_t \cos\eta\, V^2$ is the [illegible]ary normal component of the tailplane forces to provide equilibrium, the [illegible]tion of no original lack of static force balance still holding. From this equation $(BM\alpha)_t \cos\eta\, V^2$ is found. It may be fairly assumed that the normal force producing this moment is the same as would produce a moment of the same magnitude when the ship is pitched at an angle θ. Finding this force, which for consistency should be written $(mZ\alpha)_t \cos\eta V^2$, the equation corresponding with (2a) of § 12.1 is

$$R_{mz} + \{(mZ\alpha)_h \cos p_1P_2p + (mZ\alpha)_t \cos\eta\}\, V^2 = 0 \quad . \quad . \quad (2)$$

from which R_{mz} follows.

The distribution of the fin force in the normal plane can be assumed to be that of Fig. 17, so the complete set of shearing force and bending moment actions in the normal plane can now be found by the usual methods, using the moment diagram as the means of finding the aerodynamic contribution to the bending moments.

The moment due to the lateral hull component can be written $(BM\alpha)_h \cos pP_2p_3 V^2$ and the equation to be satisfied is

$$(BM\alpha)_h \cos pP_2p_3 + (BM\alpha)_t \cos(\pi/2 - \eta) = 0 \quad . \quad . \quad (3)$$

by means of which the moment due to the tailplane forces in the lateral plane is found. With the normal component found previously, it is possible if desired to find $(BM\alpha)_t V^2$ and η.

$(mZa)_t \cos(\pi/2 - \eta) V^2$ is found in a way similar to $(mZa)_t \cos \eta V^2$ or can be deduced directly from it by multiplying by $\tan \eta$ if η has been found. It is then possible as the equation

$$R_{mY} + \{ (mZa)_h \cos p P_2 p_3 + (mZa)_t \cos(\pi/2 - \eta) \} V^2 = 0 \quad . \quad . \quad (4)$$

can be solved to proceed to shearing and bending actions in the lateral plane.

For longitudinal forces, the equation is

$$R_{mX} + (mX\alpha + mX_c\alpha) V^2 = 0 \quad . \quad . \quad . \quad . \quad (5)$$

replacing equations 1*a*, §12.1 and 1, §12.2. In representing this equation and 1, 2, 3, 4 above, tabularly it is emphasised that the terms $(W - L) \sin \theta$, $(W - L) \cos \theta$, $Wh_w \sin \theta$, $Wh_L \sin \theta$ and $(W - L) c \cos \theta$ do not appear because total actions on the ship are the concern of the equations; as was explained in §12.1 however, they will appear as force distributions having no force resultant or moment in the tables.

§12.4. *Cases 85-88*

The maximum amount of false lift that can occur at the mast may be calculated from the paper quoted in §3.32. The balancing forces due to the ground weights keep the trim practically horizontal, and the case is dealt with like the earlier static cases.

§13. Cases 89-104

If the thrust is assumed to be a maximum when the engines are started and the ship is about to move, the resulting actions conveniently represent one end of a series of conditions at the other end of which are the Cases 17-32. While in practice there may be no attempt made to prevent any pitching of the ship due to the moment of the longitudinal forces, the description of Cases 17-32 requires for equilibrium a movement of weight in the ship, in accordance with the second of the equations of equilibrium

$$m (dV/dt) + T = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$m (dV/dt) h + T\tau + M = 0 \quad . \quad . \quad . \quad . \quad (2)$$

where dV/dt is the initial acceleration, and h is the distance of the centre of mass below the C.B.; dV/dt is found from equation (1). The masses in the ship are collected at frame positions and for any bay of the ship the total longitudinal force is the algebraic sum of the increments of $m (dV/dt)$ and T at frames ahead or behind the bay. The increments of the two moments of equation (2) are incorporated with the bending moments due to the static conditions modified from Cases 1-16, just as the moments represented by $Wh_w \sin \overline{\alpha + \theta}$, $Lh_L \sin \overline{\alpha + \theta}$ and $T\tau$ in §9.36 were incorporated.

§14. Cases 105-120

Deceleration tests have shown that longitudinally an airship can have a virtual mass of as much as 1.2 times its true mass. If the engines are suddenly stopped when the ship is travelling at full speed, the initial deceleration will be the greatest and it is given by

$$(mX + mX_c) V^2 + (m + m_v) (dV/dT) = 0 \quad . \quad . \quad . \quad (1)$$

where m_v is the addition due to the virtual mass effect.

To retain the true horizontal position of the ship, some redistribution of weight is needed. This is determined by

$$(mX_c) V^2 \tau + m (dV/dt) h + M = 0 \quad . \quad . \quad . \quad . \quad (2)$$

When the distribution is fixed according to the requirements of this equation (which assumes symmetry about the ship's axis for the virtual mass m_v), the distribution of longitudinal forces can be obtained. The ship's masses may be collected at frame positions, and the virtual mass, which can be supposed to make its effect felt by the way of the outer cover, can be assumed to have a distribution proportional to that of the ship's hull volume, and can be divided among the frames accordingly. The summation with the head resistance then follows in the usual way.

§15. Cases 121-136

In flight the elevators are moved sufficiently slowly for the ship to start pitching before the full elevator angle is attained. The accelerations in the initial stages of the motion are in consequence less severe than are implied by the rapid elevator movement assumed.

From the experimental tables the force which is suddenly applied to the tail areas can be deduced from the mZ value for o° pitch and full elevator. The moment it produces is obtained from the corresponding BM value. If m is the mass of the ship, ${}_\theta I_\Omega$ its moment of inertia about the C.B. in the vertical plane, w the velocity along OZ and Ω_θ the angular velocity about OY,

$$(mZ)\ V^2 = m\ (dw/dt) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$(BM)\ V^2 = {}_\theta I_\Omega\ (d\Omega_\theta/dt) \quad . \quad . \quad . \quad . \quad (2)$$

From these equations dw/dt and $d\Omega_\theta/dt$ can immediately be found.

The force on any mass m' in the ship, over and above its weight, is the algebraic sum

$$m'\ (dv/dt) + m'r\ (d\Omega_\theta/dt) \quad . \quad . \quad . \quad . \quad (3)$$

where r is its distance from the C.B. measured along the axis. For the purposes of illustration, the distribution of m may be considered to be proportional to the ordinates of the ship's shape. Actually, the distribution depends on the disposition of the weights, gas and air in the ship, but the volume distribution used for illustration is a fair approximation to a case where weights and lifts balance locally at the frames. In Fig. 36 the distributions of the $m'\ (dv/dt)$ forces,[32] and the $m'r\ (d\Omega_\theta/dt)$ forces are shown in correct proportions, and the net loading diagram resulting from them is drawn. The mZ forces on the fins and elevators are drawn with their resultant distant BM/mZ from the C.B.

In office calculations for these cases, tabular methods, such as have been described already for other cases, are most convenient. They are represented graphically by the diagrams of shearing force and bending moment given in Figs. 37 and 38. It will be appreciated that these inertia effects produce for positive pitching a hogging bending moment along the ship and in consequence reduce the static bending moments in the region of a deflated gasbag. Negative pitching due to elevator movement need not be considered with deflated gasbag cases, as the object of pitching at all in such cases is to obtain aerodynamic lift.

§16. Cases 137-152

For similar reasons to those given in §15, the accelerations obtained by supposing a sudden movement of the rudders are greater than can occur while the ship is getting into a turn.

In these cases

$$(mY)\ V^2 = m\ (dv/dt) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$(CN)\ V^2 = {}_\beta I_\Omega\ (d\Omega_\beta/dt) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where mY and CN are the force and moment on the ship at full rudder and no

[32] The worst value of $(dw/dt) + (rd\Omega/dt)$ is about $\frac{1}{3}g$.

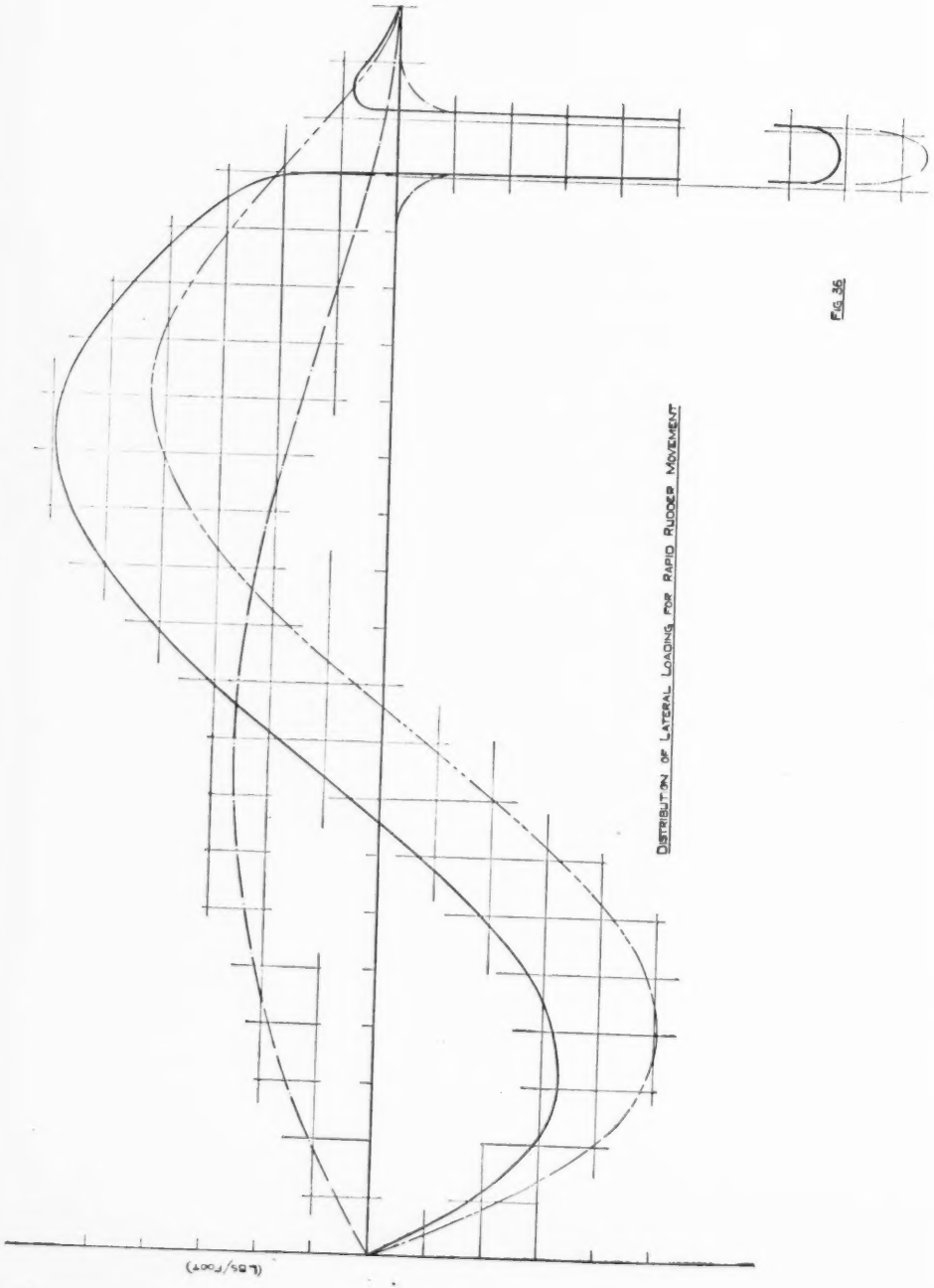

DISTRIBUTION OF LATERAL LOADING FOR RAPID RUDDER MOVEMENT
FIG 36
(LBS/FOOT)

FIG. 36.

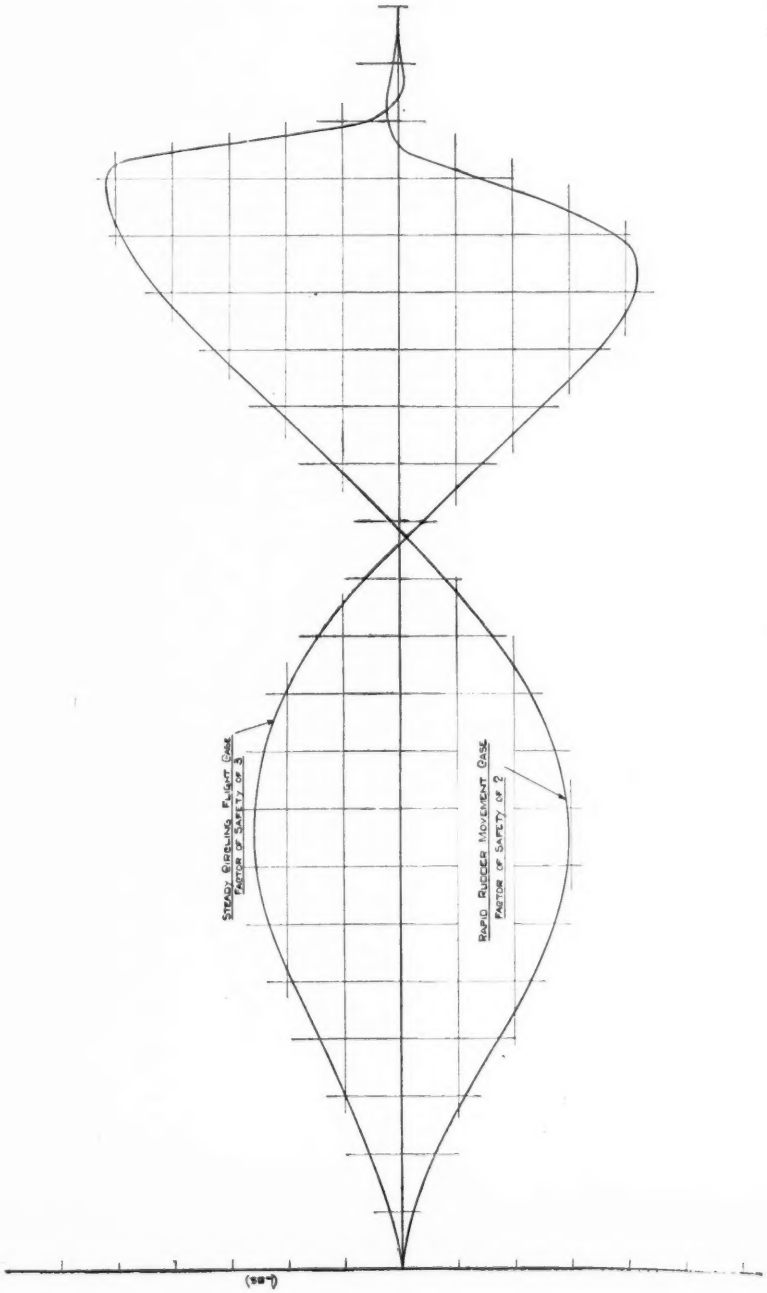


FIG. 37.
Distribution of Lateral Shearing Force in Turning.

FIG. 36.

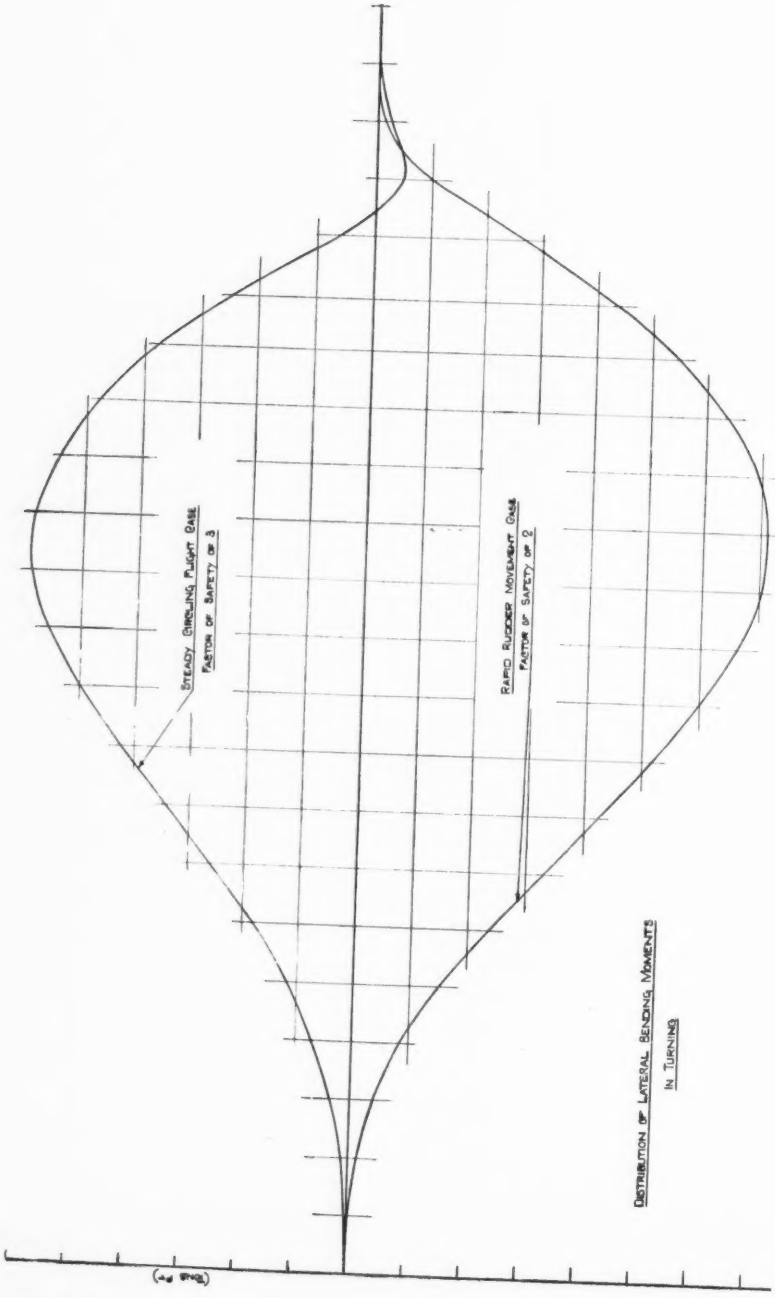

STEADY CIRCLING FLIGHT CASE
FACTOR OF SAFETY OF 3
RAPID RUDDER MOVEMENT CASE
FACTOR OF SAFETY OF 2
DISTRIBUTION OF LATERAL BENDING MOMENTS
IN TURNING

FIG. 38.

yaw, v is the velocity along OY, Ω_β the angular velocity about OZ and ${}_\beta I_\Omega$ the moment of inertia of the ship about the C.B. in the horizontal plane. .

The diagram of Fig. 36 illustrates the distribution of loading. In Figs. 37 and 38 the corresponding shearing force and bending moment diagrams are drawn with a factor of 2. In the same figures the shearing force and bending moment diagrams for steady circling flight at 75 per cent. full speed and the same rudder angle are drawn, each with a factor of 3. These diagrams are deduced from Fig. 31. The characters of the curves in Figs. 37 and 38 lead to the general conclusion that, while getting into the steady turn, the appropriately factored shearing forces and bending moments lie within the areas bounded by the curves.

§17. Cases 153-184

It is difficult satisfactorily to build up lateral force distributions on the ship in a veering or an accelerating wind. The best that can be done—and it seems sufficient—is to find the worst masthead reaction that can arise when the most rapid degrees of veering and acceleration which meteorological investigations indicate as possible, are assumed, and then to assume that the lateral loading distribution is the same as that for the steady yaw case which provides the same masthead reaction.

The method of finding the masthead reaction is dealt with fully in the paper referred to in §1.232; calculations may prove more nearly true if some value is introduced for the virtual mass of the ship.

§18. Head Resistance

It was indicated in §6.1 that the shearing and bending actions due to static forces are to some extent under the control of the designer. In addition, he wishes to avoid large concentrations of load which cannot be balanced at their points of application to the structure by other external forces. Among the most important concentrated loads are the propeller thrusts. There are more reasons than one for desiring these to be as small as possible for a given performance, but from the viewpoint of the external loading problem the reason is that they are not balanced at their points of application in the way that many large weights in the ship can be more or less balanced by large gas lift forces at the joints concerned. The balancing forces in the case of the thrusts are distributed all over the ship. These distributed forces are the drag of the ship, so that the smaller the designer can make them the more economical is the structure likely to be.

The drag of the ship is composed of the resistances due to the hull, the fins, and external cars and structures. The fin resistance is kept as small as possible by careful moulding of the longitudinal sections, but it is not the most important factor in the design of the tail planes. The parasitic resistance is made as small as possible by reducing to the minimum the external structures required, and carefully fairing them. It seems that the necessary external parts should be placed as far aft as possible, as interference in the flow round a well streamlined hull form is less harmful aft of the maximum diameter. If the dynamic lift required of the hull is not the determining factor in the design of the hull shape, then the designer has simply to seek the shape which will give the lowest head resistance and at the same time allow for economical design of the main structure and for an unexaggerated distribution of static lift. Discussion of the merits of various shapes is assisted if complete knowledge of their co-ordinates, slopes, curvatures and volumes is available. Shapes with simple mathematical equations are, therefore, of value, and the possibility of discontinuity in slope or curvature is avoided. In addition, it is possible to

calculate ordinates with confidence, and so to avoid the necessity for "laying-off" on a large scale with arbitrary curves. For this reason Appendix V has been added to this essay; it deals with the development of simple mathematical curves suitable for airship hull profiles.

§19. Conclusion

An attempt has been made in the foregoing paragraphs to indicate what external load estimations are necessary for safe airship design, and to show how they may be done. The list of Table 3 is perhaps too long for routine purposes, but, on one ship at least, it would be worth while to consider all the cases there listed, or even more, in order to learn what cases of loading may be safely ignored.

A large portion of this essay is the result of work done by the writer at the Royal Airship Works, during three years in the Department of Design and Research, which is under the direction of Lieut.-Colonel V. C. Richmond, O.B.E. He wishes to acknowledge help and advice received during this period from the Head of the Department, and from his colleagues, notably Mr. T. S. D. Collins and Miss H. M. Lyon. *For opinions expressed or implied in the essay, however, he wishes to accept sole responsibility as there are points dealt with that are still controversial, and on which Colonel Richmond and the writer's colleagues are not necessarily in agreement with him.*

Appendix I.

Note on the Speed when Circling

If in equations (1) and (2) in §10.2 acceleration derivatives are ignored,

$$mX + T - (mV^2/R)\sin\beta = 0 \quad . \quad . \quad . \quad . \quad (1)$$
$$mY - (mv^2/R)\cos\beta = 0 \quad . \quad . \quad . \quad . \quad (2)$$

From these equations,

$$mX\cos\beta - mY\sin\beta + T = 0$$

If β is the angle of yaw at the C.B. when the ship is circling with maximum rudder, then from this equation the speed at which the ship would fly if it could proceed rectilinearly with maximum rudder angle, at an angle of yaw, can be found. For R.33, with rudder angle 30°, and the corresponding β value, the speed arrived at is 79.5 per cent. of the maximum, so that the 75 per cent. figure suggested by the Airworthiness of Airships Panel is probably adequate as mX_r (§10.2) adds to the drag.

Nothing very definite on the effect of pitch in a turn can be said. Reference to Fig. 24 shows that in pitch, within limits, speed increases the greater the excess of weight over static lift. If for R.33 $(W-L)$ is taken at a round figure rather greater than the lift of its largest bag, then at 10° pitch with flight path horizontal, the speed is almost exactly 75 per cent. of the maximum.

Roughly, therefore, if a reduction on the speed in circling flight of 25 per cent. is to be expected, the lateral forces in turning will only be $(\frac{3}{4})^2$, or $\frac{9}{16}$ of what they were at 0° pitch, while the aerodynamic forces in pitch will be $(\frac{3}{4})^2$, or $\frac{9}{16}$ of what they were at 0° yaw (in these circumstances the ship could not support the large $(W-L)$ difference aerodynamically, but that is not required of it; $(W-L)$ was only introduced to get a fair figure for the speed in pitch). If it be assumed that the worst aerodynamic forces in circling flight and in pitch are of the same order, then in turning pitch we may expect resultant aerodynamic forces $(\frac{9}{16})\sqrt{2}$ or 0.8 of either of them.

APPENDIX II.

Note on Longitudinal Forces, Lateral Forces, Yawing Moments, Shearing Forces and Bending Moments due to Aerodynamic Pressure

Consider the surface of the ship between two planes perpendicular to its axis and δx feet apart. Suppose the pressure per square foot at any point on the circumferential strip between the planes is p lbs. Referring to Fig. 39, if the mean radius of the ship between the two planes is y, the total force on an element of the ship is $p \,.\, \delta s \,.\, y\delta w$.

The longitudinal force on the element is $p \,.\, \delta y \,.\, y\delta w$ and the lateral force $p\delta x \,.\, y\delta w \sin w$.

The total longitudinal force on the strip is

$$\int_{w=0}^{w=2\pi} p \,.\, dy \,.\, y \,.\, dw = y \,.\, dy \int_0^{2\pi} p\, dw$$
$$= P_x dx$$

and the total lateral force on the strip is

$$\int_{w=0}^{w=2\pi} p \,.\, dx \,.\, y dw \sin w = y \,.\, dx \int_0^{2\pi} p \sin w \,.\, dw$$
$$= P_y dx$$

where P_x and P_y are longitudinal and lateral forces per foot run of the x-axis.

The total longitudinal force up to any point X is given by

$$\int_{\text{Nose}}^{X} P_x dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The total lateral force up to any point X is given by

$$\int_{\text{Nose}}^{X} P_y dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Consider now moments about a vertical axis through the C.B. and suppose the origin of axes there. The moment of the elementary longitudinal force is $p\delta y \,.\, y\delta w \,.\, y \sin w$ and the total moment from the whole circumferential strip is

$$y^2 dy \int_0^{2\pi} p \sin w \,.\, dw$$
$$= y P_y dy \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The moment of the elementary lateral force is $p\delta x \,.\, y\delta w \sin w \,.\, x$ and the total moment from the strip is

$$yx dx \int_0^{2\pi} p \sin w dw$$
$$= x P_y dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

So the total yawing moment due to the strip is

$$P_y (x dx + y dy) = P_y d \{ \tfrac{1}{2} (x^2 + y^2) \} \quad . \quad . \quad . \quad (5)$$

Ignoring skin friction, P_x and P_y represent longitudinal force distribution and lateral force distribution as plotted in Figs. 15 and 7, expressions (1) and (2) represent longitudinal force and shearing force as shown in Figs. 16 and 8, while

$$mX = \int_{\text{Nose}}^{\text{Tail}} P_x dx \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$mY = \int_{\text{Nose}}^{\text{Tail}} P_y dx \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$CN = \int_{\text{Nose}}^{\text{Tail}} P_y d \{ \tfrac{1}{2} (x^2 + y^2) \} \quad . \quad . \quad . \quad . \quad (8)$$

In Fig. 6 P_y is shown plotted on $\frac{1}{2}(x^2+y^2)$ if x is measured from the C.B.

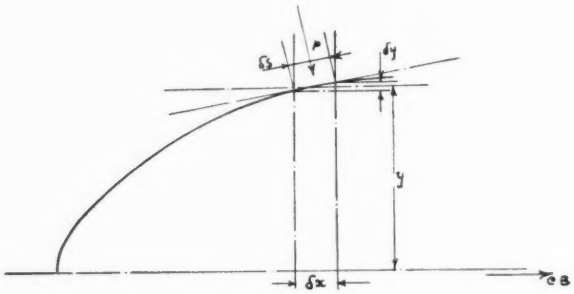
Consider a point N on the ship's axis distant d from the C.B. Then bending moment at N

$$= \int_A^N pd \{ \tfrac{1}{2} ((x-d)^2 + y^2) \}$$

$$= \int_A^N pd \{ \tfrac{1}{2} ((x^2+y^2) - dx + d^2/2) \}$$

$$= \int_A^N pd \{ \tfrac{1}{2} (x^2+y^2) \} - d \int_A^N pdx$$

$$= [\text{area } ABS \text{ (Fig. 6)}] - d [\text{shear at } N \text{ (Fig. 9)}].$$



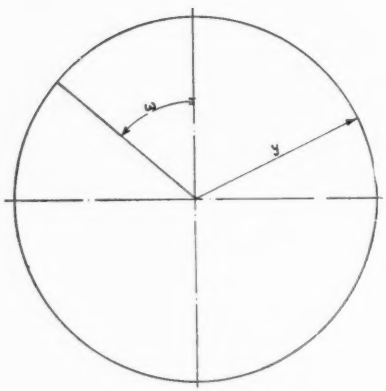


FIG. 39.

Using the notation of §7.4 the bending moment is given by

$$[\text{area } ABN \text{ (Fig. 6)}] - (c-x) [\text{ordinate at } N \text{ (Fig. 9)}].$$

For examples of aerodynamic loading at the mast, the moment diagram is drawn on $\frac{1}{2}(x^2+y^2)$ as distinct from $\frac{1}{2}((c-x)+y^2)$. The diagram has no longer the characteristic shape of Fig. 6; it is like a distorted lateral loading distribution diagram. There is a corresponding theorem for obtaining the bending moment from it.

APPENDIX III.

Coefficients in the Expression for Pressure at a Point on a Spheroid, with reference to a Circling Flight of R.33

An attempt has been made to build up the distribution of pressure on the hull of R.33 in circling flight. It is not proposed to give the resulting curves here, but it is thought that perhaps the constants necessary for the evaluation of the expression[34] for pressure at a point are worth recording.

The whole curved portion of the profile from the nose to the beginning of the parallel portion of the ship was taken as elliptical. The semi-major axis of this ellipse is 56 metres and the semi-minor axis is 12 metres. Hence $a/b = 4.6667$. The quantities L, M and N (§§7.2, 10.3) are then 0.06539, 0.8836 and 0.7735 respectively. The calculation was done for 5° helm, when the yaw at the C.B. was found to be 4.5°. The local yaw at the centre of the nose spheroid was then 2° 54′, and the radius of turn there was 3563.5 feet. The expressions for a_0, a_1, a_2, a_3, b_1, b_2, b_3, c_2, c_3, are given below, and their values are tabulated for the yaw conditions just described. To obtain them for any other yaw β, constant multipliers are required for the values of each. These are given with the tables as ratios denoted by K.

$$a_0 = \tfrac{1}{2}\rho V^2 \cos^2\beta \{ 1 - [(1+L)^2 \sin^2\psi]/[\sin^2\psi + (b^2/a^2)\cos^2\psi] \}$$

$$a_1 = -\tfrac{1}{2}\rho V^2 [2(b/a)(1+L)(1+M)\sin\beta\cos\beta\sin\psi\cos\psi]/[\sin^2\psi + (b^2/a^2)\cos^2\psi]$$

$$a_2 = \tfrac{1}{2}\rho V^2 \sin^2\beta \{ 1 - [(b^2/a^2)(1+M)^2\cos^2\psi]/[\sin^2\psi + (b^2/a^2)\cos^2\psi] \}$$

$$a_3 = \tfrac{1}{2}\rho V^2 \sin^2\beta \{ 1 - (1+M)^2 \}$$

$$\tfrac{1}{2}b_1\Omega = \tfrac{1}{2}\rho V^2 (b/R)\cos\beta\sin\psi$$
$$\{ [(1+L)(1+e^2N\cos 2\psi)]/[\sin^2\psi + (b^2/a^2)\cos^2\psi] - 1 \}$$

$$\tfrac{1}{2}b_2\Omega = \tfrac{1}{2}\rho V^2 (a/R)\sin\beta\cos\psi$$
$$\{ [(b^2/a^2)(1+M)(1+e^2N\cos 2\psi)]/[\sin^2\psi + (b^2/a^2)\cos^2\psi] - 1 \}$$

$$\tfrac{1}{2}b_3\Omega = \tfrac{1}{2}\rho V^2 (a/R)\sin\beta\cos\psi \{ (1+M)(1+e^2N) - 1 \}$$

$$c_2\Omega^2 = \tfrac{1}{2}\rho V^2 (a^2/R^2) \{ \cos^2\psi + (b^2/a^2)\sin^2\psi$$
$$- [(b^2/a^2)(1+e^2N\cos 2\psi)^2]/[\sin^2\psi + (b^2/a^2)\cos^2\psi] \}$$

$$c_3\Omega^2 = \tfrac{1}{2}\rho V^2 (a^2/R^2)\cos^2\psi \{ 1 - (1+e^2N)^2 \}$$

ψ (°)	$a_0/\frac{1}{2}\rho V^2$
0	.9974
5	.8355
10	.5404
20	.15645
30	.00225
40	−.0676
50	−.0995
60	−.1178
70	−.1281
80	−.1332
90	−.1349

$K = \cos^2\beta/\cos^2 2° 54'$

[34] §10.4.

$a_1/\frac{1}{2}\rho V^2$		
0	0	
5	−.07096	
10	−.09954	
20	−.08870	
30	−.06612	
40	−.04862	
50	−.03532	
60	−.02471	
70	−.01573	
80	−.00765	
90	0	
		$K=\sin 2\beta/\sin\ 5^\circ\ 48'$

$a_2/\frac{1}{2}\rho V^2$		
0	−.0065245	
5	−.0052245	
10	−.0028575	
20	+.0002215	
30	.0014605	
40	.0020043	
50	.0022760	
60	.0024236	
70	.0025056	
80	.0025476	
90	.0025605	
		$K=\sin^2\beta/\sin^2\ 2^\circ\ 54'$

$a_3/\frac{1}{2}\rho V^2$		
	.006524	
		$K=\sin^2\beta/\sin^2\ 2^\circ\ 54'$

$b_1\Omega/\frac{1}{2}\rho V^2$		
0	0	
5	.06466	
10	.088745	
20	.07238	
30	.04555	
40	.02456	
50	.00902	
60	−.002238	
70	−.00993	
80	−.014425	
90	−.015905	
		$K=\cos\beta/\cos\ 2^\circ\ 54'$

$b_2\Omega/\frac{1}{2}\rho V^2$	
0	.011864
5	.009406
10	.004941
20	−.000689
30	−.002638
40	−.003111
50	−.002937
60	−.002422
70	−.001709
80	−.000881
90	0

$K = \sin\beta/\sin\ 2^\circ\ 54'$

$b_3\Omega/\frac{1}{2}\rho V^2$	
0	.011863
5	.011818
10	.011682
20	.011146
30	.010272
40	.009088
50	.007625
60	.005932
70	.004057
80	.002059
90	0

$K = \sin\beta/\sin\ 2^\circ\ 54'$

$c_2\Omega^2/\frac{1}{2}\rho V^2$	
0	−.005371
5	−.004207
10	−.002108
20	.000462
30	.001221
40	.001258
50	.001016
60	.000692
70	.000393
80	.000187
90	.000114

$K = 1$

$c_3\Omega^2/\frac{1}{2}\rho V^2$	
0	−.005372
5	−.005331
10	−.005210
20	−.004744
30	−.004029
40	−.003152
50	−.002220
60	−.001343
70	−.000628
80	−.000162
90	0

$K = 1$

Appendix IV.

Airship Shape

In §4.6 of his R.38 Memorial Prize Essay,[35] Dr. Jones gives some "broad conclusions" deduced from experiments on model shapes. He mentions that a shape of fineness ratio 4.6 : 1 can be produced with a resistance coefficient of 0.007, whereas the resistance coefficient for R.33 was 0.011. The forward curved portion should be, it is stated, at least "two diameters" long, which is to say, that the maximum diameter should be at least 0.435 of the total length from the nose for a fineness ratio of 4.6 : 1. It has not been shown that so low a fineness ratio as this is essential for low resistance, and if the particular 4.6 : 1 shape U.721 is modified, by multiplying its ordinates by a constant factor, into a 5.4 : 1 shape the resistance coefficient does not appreciably alter;[36] but an exact and comprehensive inquiry into the question of the effect of fineness ratio is really required.

It is generally agreed that a good hull profile should decrease in curvature from the nose to the tail. This decrease should certainly be continuous, but there is at present no evidence to indicate how the decrease should be made. It will be helpful, no doubt, to consider in this connection shapes to which mathematical equations can be attached, so that the characters of different curvatures can be exactly described. Such shapes are useful to designers in that ordinates are exactly calculable, together with volumes and curved lengths if the equation is easily integrable.

The manner of developing series of such curves, which is to be described, was suggested by the present writer in 1924, when the shape of the R.101 was under discussion, and numerous shapes were produced whose geometrical characteristics could, in consequence, be easily compared. A brief reference to the work has been made by Lieut.-Colonel Richmond in a paper to the Inst. of N.A.[37]

The method of obtaining suitable curves with simple equations was suggested by the idea that if during the generation of the ellipse, the minor axis were supposed to vary according to some simple law, the nose could be shortened and the tail extended to produce finally a shape of the type called "streamline."

It is clear that such a scheme consists in multiplying the ordinates of a circle (or of an ellipse) by some other curve, but the original idea of varying the minor axis of an ellipse is found useful in producing certain desired proportions, and facilitates comparison of the curves with an ellipse of the same fineness ratio. The equation to an ellipse whose major axis lies between $x=0$ and $x=2a$ is

$$y_e = \pm (b/a)\sqrt{(2ax - x^2)} \quad . \quad . \quad . \quad . \quad (1)$$

where $2b$ is the minor axis and the desired fineness ratio is a/b.

If now the minor axis is allowed to be a suitable function of x, a curve of shape is given by

$$\begin{aligned} y_1 &= \pm (f(x)/a)\sqrt{(2ax - x^2)} \\ &= \pm (f(x)/b)\, y_e \quad . \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

where y_e is the ordinate of the basic ellipse of equation (1) and b its semi-minor axis.

Suppose now that the maximum diameter of the shape represented by (2) occurs at $x=X$. Then

$$(dy/dx)^{x=X} = 0$$

$$i.e.,\ (f'(x)\, y_e + f(x)\, y'_e)^{x=X} = 0$$

$$\text{or } f'(x)^{x=X}\sqrt{(2aX - X^2)} + f(X)(a - X)/\sqrt{(2aX - X^2)} = 0 \quad . \quad . \quad (3)$$

[35] *loc. cit.*
[36] R. & M. 607 and T.2034.
[37] *loc. cit.* §6.1.

From (3) X may be determined, and if Y_1 is the value of y_1 at X

$$Y_1 = \pm f(X)/a \sqrt{(2aX - X^2)}$$

But if y is the ordinate of the final curve, of major axis $2a$ and fineness ratio a/b equal to that of the basic ellipse, b is the maximum ordinate, and

$$y = \pm (b/Y_1)(f(x)/b)\, y_e$$
$$= \pm (f(x)/Y_1)\, y_e$$

The curve by which the original ellipse must be multiplied is therefore $y = (f(x)/Y_1)$. Simple mathematical forms may be chosen for $f(x)$. The choice must be a function continuous in dy/dx and d^2y/dx^2 to ensure that the final shape curve is continuous in the same way. If $f(x)$ is zero for $x = 2a$ then the shape will have a pointed tail; if it is zero for $x = 0$ a pointed nose will result.

f (x) a linear function

One possibility is to employ a straight line as the multiplying curve. Then

$$f(x) = kx + c \qquad . \quad . \quad . \quad . \quad . \quad (5)$$

Referring to Fig. 40*a*, it will be seen that the shape of the final curve will depend on the value of m, so equation (5) is best expressed as

$$f(x) = k\{x - (2 + m)a\} \qquad . \quad . \quad . \quad . \quad (5a)$$

where k is any constant.

If for $a = 10$ and a fineness ratio of 5.5 : 1, $m = 0$, the result is the shape 1 in Fig. 41. The lesson to be drawn is that m should have some positive value. Shape 2 has $m = 1$.

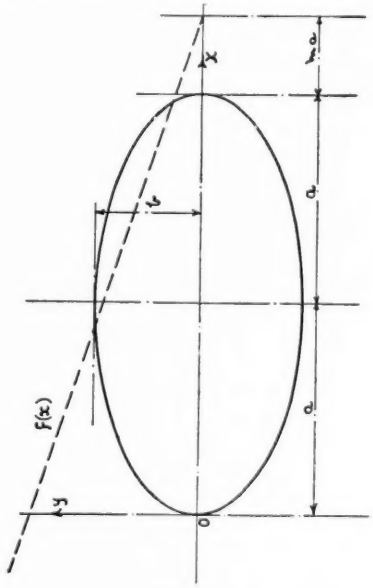


FIG. 40*a*.

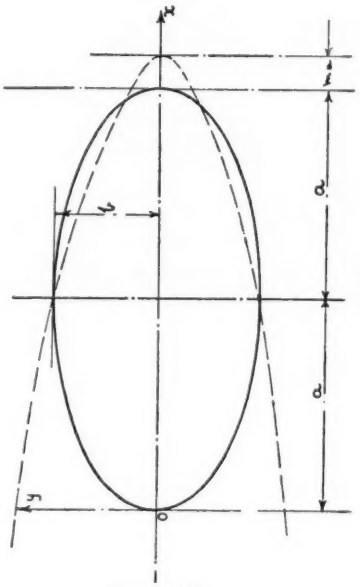


FIG. 40*b*.

f (x) a parabolic function

Referring to Fig. 40*b* with the same notation as before

$$[f(x)]^2 = k\{(2 + m)a - x\} \qquad . \quad . \quad . \quad . \quad (6)$$

Shapes 3 and 4 give the results of assuming $m = 0$ and $m = 1$.

f (x) an elliptic function

In Fig. 40*c*, n defines the position of the minor axis of the multiplying ellipse, and

$$[f(x)]^2 = k^2 \{ 1 - (x+na)^2/(2+m+n)^2 a^2 \} \quad . \quad . \quad (7)$$

Shapes 5, 6 and 7 show the results of making simultaneously

$$\left\{ \begin{matrix} n=0 \\ m=0 \end{matrix} \right\} \quad \left\{ \begin{matrix} n=1 \\ m=0 \end{matrix} \right\} \quad \left\{ \begin{matrix} n=1 \\ m=.5 \end{matrix} \right\}$$

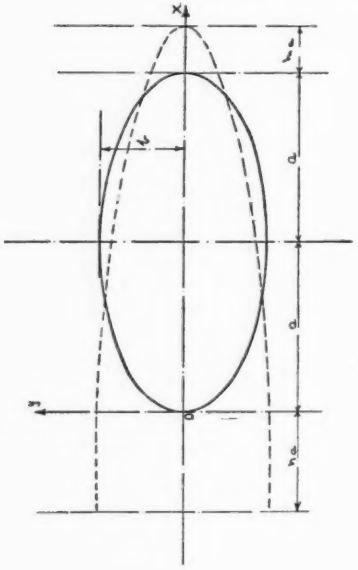


FIG. 40*c*.

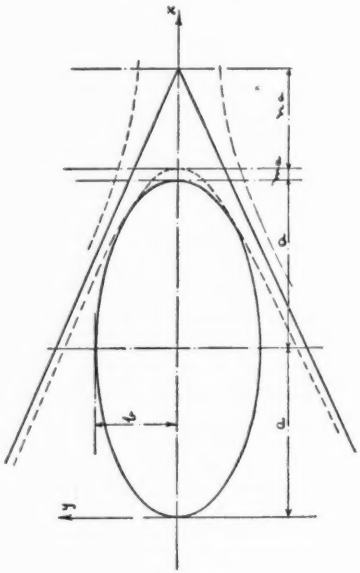


FIG. 40*d*.

f (x) a hyperbolic function

Referring to Fig. 40*d*,

$$[f(x)]^2 = k^2 [\{ (2+m+n)\, a - x \}^2 / n^2 a^2 - 1]$$

Shapes 8 and 9 show the results of making simultaneously

$$\left\{ \begin{matrix} n=1 \\ m=0 \end{matrix} \right\} \quad \left\{ \begin{matrix} n=1 \\ m=.5 \end{matrix} \right\}$$

The most successful shapes are those following the adoption of elliptic forms for $f(x)$. In shape 3 curvature decreases continuously from nose to tail and purely on the basis of head resistance this is probably the best shape. The relative importance of an increase of curvature in the region of the fins is not however fully known. In practice, the turbulence in the flow there may make the shape in the fin region quite unimportant and other considerations than head resistance may require a tail less fine than that of shape 3. For instance, if it is necessary to cater for heavy weights aft, stout tails may be necessary to provide adequate gas lifts, and shapes like 4 or 7 may be in the circumstances desirable. Also good block coefficients demanded by limitation of shed accommodation, may be an important factor in deciding upon a particular shape when shape 3 would perhaps be discarded in favour of others of greater head resistance.

It is not suggested that shape 3 has definitely a lower resistance than all others, but it is sufficiently good to be used as a basis for an experiment to determine the best position for the maximum diameter. If m is kept constant, the maximum diameter moves forward as n increases positively, so tests on a series of models of shapes built up with $f(x)$ defined as in equation (7) with $m=0$ and n varying from $+2$ to -2, should be instructive.

The foregoing paragraphs have described a method of shape production which is fairly elastic, and in general the fore portions of the curves are sufficiently close to a true ellipse to allow the hydrodynamical theory described in §7.2 to be applied. It is possible, however, to build up a shape continuous in dy/dx and d^2y/dx^2 in which the profile from the nose to the maximum diameter is a true ellipse, and in which the radius of curvature increases continuously to the tail. If the shape is of length l, the maximum diameter is fixed at a distance d from the nose, and d^2y/dx^2 is made to vary linearly, the ordinates of the after portion are given by

$$y=(m/6)\,x^3+(C/2)\,x^2+Dx+E \quad [38]$$

where

$$m/6=(b/2d^2)\,(l^2-2dl-d^2)/(l-d)^3$$
$$C/2=-\tfrac{1}{2}\,(b/d^2+md)$$
$$D=b/d+md^2/2$$
$$E=b/2-(m/6)\,d^3$$

A linear variation for d^2y/dx^2 may not give the best type of variation of curvature, and many different curves based on different laws of curvature variation can be constructed. An interesting one is the involute of an involute, in which the radius of curvature increases progressively from nose to tail. The coordinates of this curve[39] are given by

$$x=(a\cos\psi+a\psi\sin\psi-\tfrac{1}{2}a\psi^2\cos\psi)-a$$
$$y=(a\sin\psi-a\psi\cos\psi-\tfrac{1}{2}a\psi^2\sin\psi)$$

where ψ is angle representing the amount by which the original circle has "unrolled." If the curve is supposed to start with $\psi=0$ the shape will be complete at $\psi=\tan^{-1}2\psi/(2-\psi^2)$.

The number of sides an airship has appears to have only a very small effect on head resistance. This is chiefly because the surface area of a ship of polygonal section is very little greater than that of the ship of circular section and the same volume. Table 7 is appended to show how these areas compare as the number of sides increases. Less is known of the effect of the number of sides on normal or lateral forces, and experiments to determine it would prove very useful.

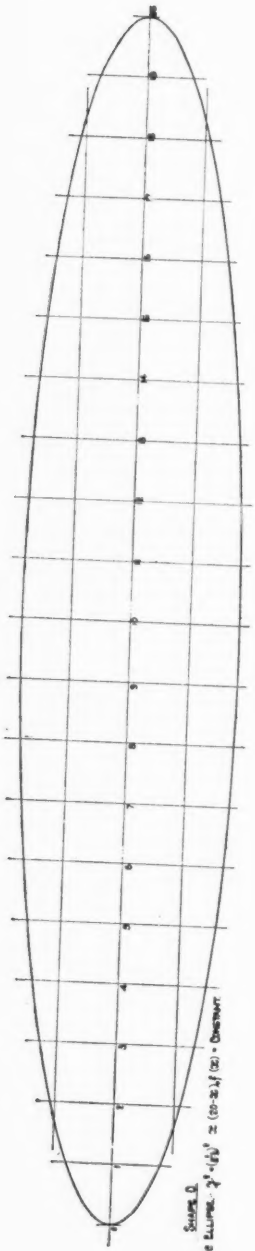
TABLE 7.

Number of sides = n	Ratio of surface area of ship with n sides to that of ship of circular section of the same volume.
5	1.1260
10	1.0283
15	1.0122
20	1.0066
25	1.0040
30	1.0029

From the head resistance standpoint, the question of fineness ratio is much more important. Experiments on the same fundamental shape, modified to different fineness ratios, are really needed, and it is thought that use of one of the mathematical shapes given might be advantageous.

[38] Plotted as shape 10 for $d=4$, $l=10$.

[39] The natural fineness ratio is 2.5. Shape 11 is the natural curve modified by a constant multiplier.



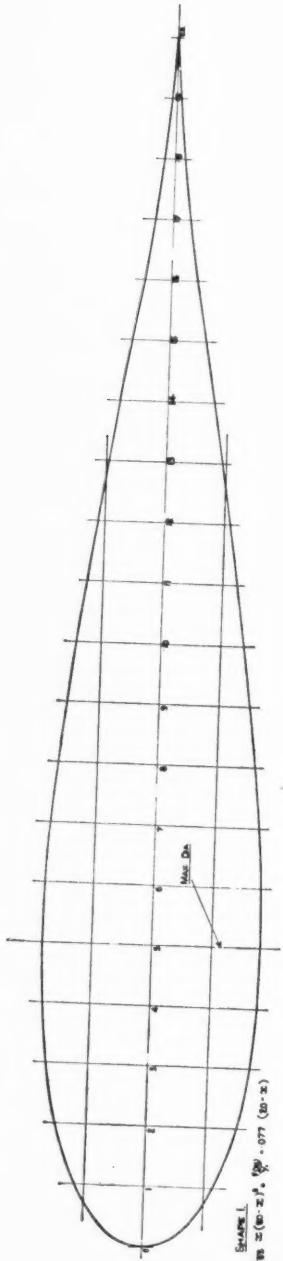
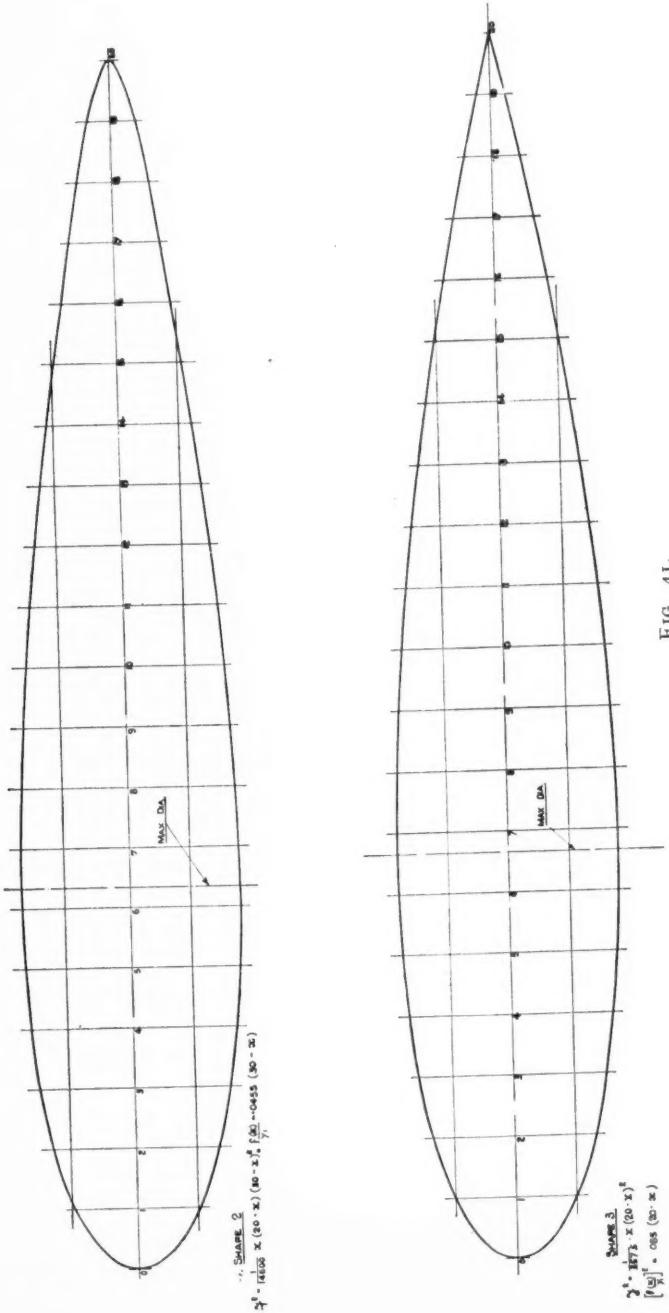


FIG. 41.



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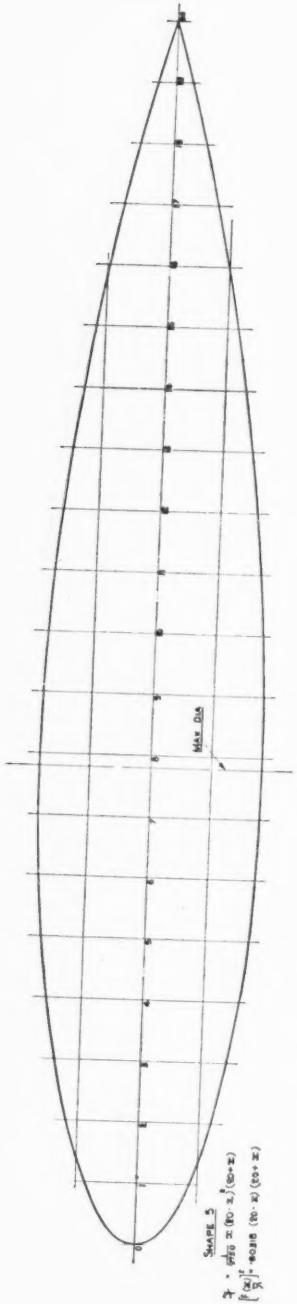

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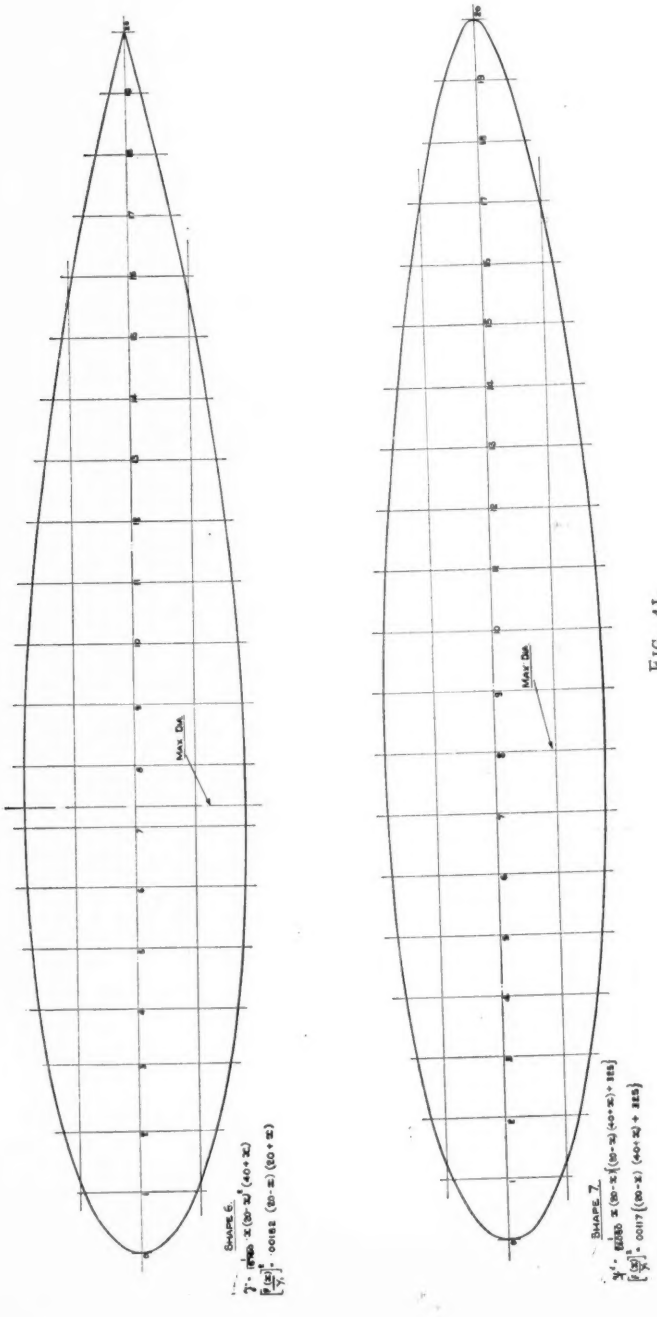

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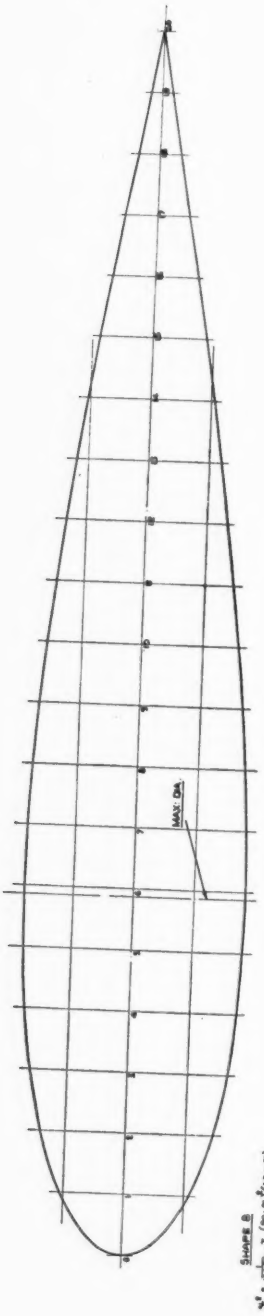

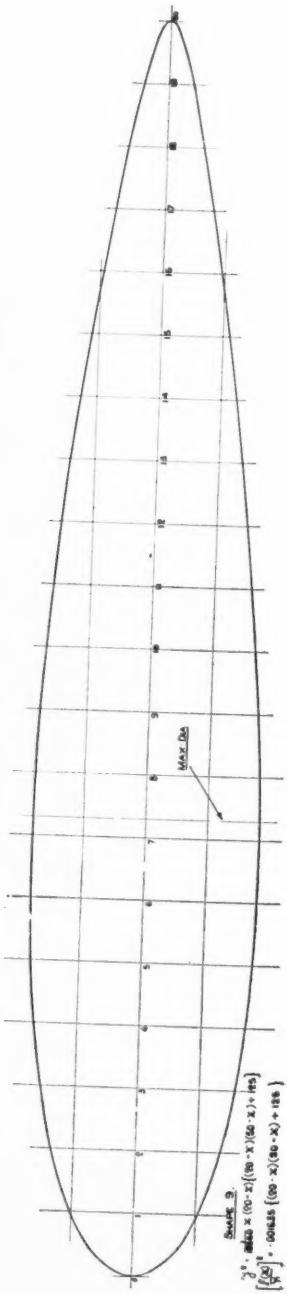

FIG. 41.

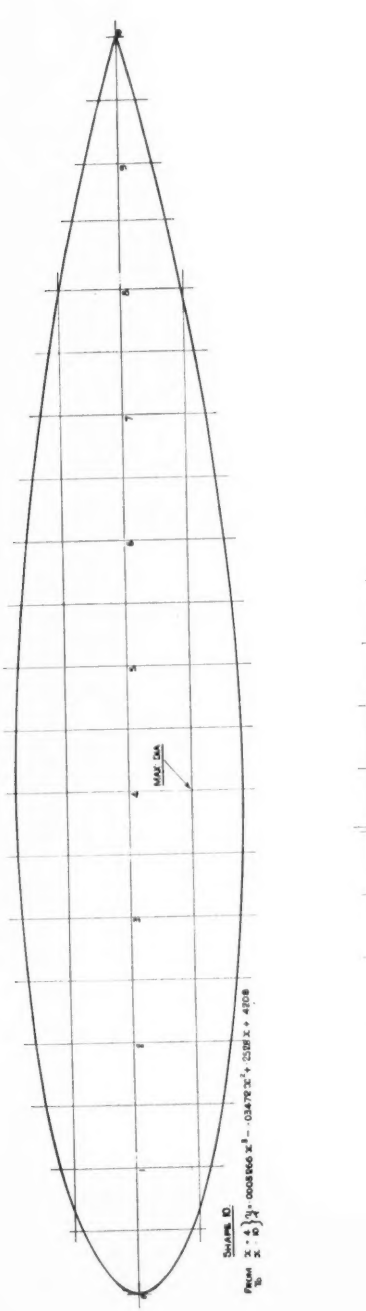

FIG. 41.

SUMMARY

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- §1.1. The Possible Combinations of the Components.
- §1.2. The Variations of the Components.
- §1.21. Component W (Weight Component).
- §1.22. Component L (Lift Component), Table 1.
- §1.23. Component A (Aerodynamic Component).
- §1.231. In Free Flight.
- §1.232. Ship at Mast.
- §1.233. Ship being Moored.
- §1.234. Ship on the Ground.
- §1.235. Variation of Component A, Table 2.
- §1.24. Component T (Thrust Component).
- §1.25. Component R (Reaction Component).

§2. Factors of Safety.

§3. The Systems of External Loading to be Considered in Design.
- §3.1 Ship in Shed.
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- §3.4. Table 3 (Systems of External Forces to be Considered in Design).

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§5. Estimation of Static Loading.

§6. Cases 1-16[40] (Cases of "Static" Loading, Ship Stationary).
- §6.1. Case 1, Table 4.
- §6.2. Case 2.
- §6.3. Cases 3 and 4.
- §6.4. Case 5.
- §6.5. Cases 6, 7 and 8.
- §6.6. Cases 9-16.

§7. Details of Component A for Unpitched and Pitched Flight with no Yaw.
- §7.1. Experimental Data Required.
- §7.2. Curves of Normal Force Distribution.
- §7.3. Curves of Variation of Shearing Force.
- §7.4. Curves of Variation of Bending Moment.
- §7.5. Curves of Pressure Distribution, Table 5.
- §7.6. Curves of Longitudinal Force Distribution.
- §7.7. The Forces due to the Horizontal Fins and Elevators.
- §7.8. Forces due to the Cars.
- §7.9. "Stepping" of Diagrams.

§8. Details of Component T.

§9. Cases 17-52[41] (Cases of Pitched and Unpitched Flight).
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- §9.2. Cases 17-32 (Pitch 0°).
- §9.3. Cases 33-44 (Positive Pitch).
- §9.33. Solution of Equations of Equilibrium.
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- §9.38. A Simpler Solution.
- §9.4. Cases 45-52 (Negative Pitch, Superheating).

[40] See Table 3.

§10. Details of Component A for Circling Flight at 0° Pitch.
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§10.2. Equations of Equilibrium.
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§10.5. Longitudinal Loads.
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§11.2. Shearing Forces, Bending Moments, and Longitudinal Forces.
§11.3. Rolling and Pitching Moments in Circling Flight.
§11.4. Centrifugal Force of Air contained in Ship.
§12. Cases 69-88[41] (Cases for the Ship at the Mast).
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§12.2. Yaw at the Mast.
§12.3. Pitch and Yaw at the Mast.
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§13. Cases 89-104[41] (Initial Acceleration).
§14. Cases 105-120[41] (Deceleration).
§15. Cases 121-136[41] (Rapid Elevator Movement).
§16. Cases 137-152[41] (Rapid Rudder Movement).
§17. Cases 153-184[41] (Varying Winds while Ship is Moored).
§18. Head Resistance.
§19. Conclusion.
Appendix I. Note on the Speed when Circling.
Appendix II. Note on Forces and Moments due to Aerodynamic Pressures.
Appendix III. Coefficients in the Expression for Pressure at a Point on a Spheroid.
Appendix IV. Airship Shapes, Table 7.

[41] See Table 3.

PROCEEDINGS

TENTH MEETING, SECOND HALF, 64TH SESSION

In the Chair:

THE PRESIDENT, COLONEL THE MASTER OF SEMPILL

A meeting of the Royal Aeronautical Society was held in the Lecture Theatre of the Royal Society of Arts, 18, John Street, Adelphi, W.C.2, on Thursday, April 25th.

THE PRESIDENT: The paper by Squadron Leader C. L. Scott would be found to be an interesting one, following as it did the recent account before the Society by Captain Cave-Browne-Cave of the cruise by a number of flying boats to the Far East. Squadron Leader C. L. Scott had specialised on flying boats for many years. He had participated in the various cruises to the Baltic. Of the recent cruise with the Under-Secretary of State for Air, it was fair to say that, apart from the fact that there had been engine trouble, which was no fault of Squadron Leader Scott, the cruise was a success, notwithstanding the delays that took place from that cause. It was interesting to note that had a land machine been used and similar engine troubles been met with he certainly would not have come home with the machine intact. It was only because the machine was a flying boat that he was enabled to make forced landings in that inhospitable region and effect the necessary repairs, and, after delays incidental to fitting a spare engine, bring the machine back to its base in this country. He congratulated Squadron Leader Scott on that achievement.

BY FLYING BOAT TO INDIA

BY

SQUADRON LEADER C. L. SCOTT, D.S.C.

I am indebted to the Air Ministry for permission to read this paper to you, and the views which I have tried to express represent my own personal opinion and are not necessarily the views of the Air Ministry.

Objects of the Flight

The objects of this flight were (1) to enable the Under-Secretary of State for Air and the Director of Equipment, Royal Air Force, to inspect as many as possible of the Royal Air Force Units in Egypt, Iraq, India and Malta, and (2) to get some information on the behaviour of a large flying boat operating under tropical conditions away from its base.

The Aircraft

The flight was carried out in the Blackburn "Iris II." flying boat with duralumin hull, fitted with three Rolls-Royce Condor III.A engines and four-bladed wooden propellers painted with cellulose paint. Dual control was fitted and the two pilots sat side by side.

The fuel was carried in three tanks attached to the under side of the top centre section over the three engines. Each of these tanks had a capacity of 300 gallons and was fitted with a fuel contents gauge on the underside.

Pipes were led from each of these tanks to a central junction box in the lower centre section and from this junction box three pipes were led to the three engines.

Cocks, capable of being operated from inside the hull, were fitted to these six pipes and thus complete control of the fuel supply was obtained.

It was found in practice that, with all cocks on, the fuel did not drain equally from the two outboard tanks and the loss of lateral trim soon became noticeable. This was easily rectified by temporarily shutting off the supply from the tanks which contained the less fuel.

A biplane tail was fitted and there were three rudders, a servo rudder being fitted to the centre rudder only. The servo gear could be put out of action by means of a clutch operated from the pilot's cockpit.

Two elevators were fitted, the upper and larger one being operated in the ordinary way by the control column, and the lower one being operated by means of a lever in the pilot's cockpit. This fixed elevator took the place of an adjustable tail plane.

A small duralumin dinghy weighing about 80 lbs. was carried in an inverted position in the lower centre section. This dinghy carried five people and proved to be very useful.

Personnel

The crew consisted of two pilots, two fitters, one W/T. operator and one rigger.

The passengers were Sir Phillip Sassoon, Under-Secretary of State for Air, Air Commodore Longmore, Director of Equipment, and a servant.

Loading

The normal service load of the " Iris " with 680 gallons of fuel is 27,400 lbs.

On this flight, with full fuel (900 gallons), the gross load was 30,350 lbs.

For most of the stages only 700 gallons of fuel were carried, and this reduced the gross load to 28,850 lbs.

The maximum gross load at which the " Iris " has been flown is 33,000 lbs.

The Flight

The " Iris " left Felixstowe on September 27th for Plymouth. Refuelling was carried out on arrival and the next day was spent in having a general look over the machine and engines. The passengers joined the " Iris " early on the morning of September 29th and we left for Hourtin, near Bordeaux, at 06.55 hours.

A certain amount of low cloud was encountered in the Channel and there were some very unpleasant hailstorms in the neighbourhood of Ushant.

After Ushant was passed the weather improved and we had practically no trouble from weather for the remainder of the outward trip.

We landed near the French Seaplane Station on the lake at Hourtin and took in 400 gallons of fuel in just over an hour.

Berre, near Marseilles, was reached just before dark the same day.

From here Aboukir was reached in three days, via Naples and Athens.

At Aboukir the passengers left the " Iris " and proceeded to Khartoum and then across the desert route to Iraq by land aircraft.

The " Iris " spent six days at Aboukir during which time everything was carefully inspected. The only defect found in the engines was excessive wear

in several of the bushes in one rocker box. The complete rocker box was removed and a new one fitted.

We left Aboukir for Alexandretta on October the 8th, and it was on this occasion that difficulty was first experienced in taking-off with no wind.

The tanks had been filled up to 800 gallons and the gross load was 28,820 lbs.

Two attempts were made to get off, but on neither occasion could an air speed of 40 knots be exceeded. One hundred gallons of fuel were then taken out, reducing the load to 28,070 lbs.

Another failure was experienced, due to a slight swell which was running. Each time the fore part of the hull struck a swell the speed was noticeably reduced.

The final attempt was made close in shore to avoid the swell and here we got off after a very long run.

Our intention was to leave Alexandretta on October 10th. We climbed to a height of 5,500 feet, but we were unable to get through the mountain pass owing to clouds and rain storms.

The weather was better the next day and we went through the pass at a height of 4,000 feet and picked up the Euphrates at Meskene after flying over the desert for about 1½ hours. The river was followed and a height of over 4,000 feet was maintained because below this height the air was uncomfortably bumpy.

We landed on the large lake of Habbaniya, about 60 miles short of Baghdad, shortly after mid-day.

This was done as it was thought that it would be very difficult to bring the "Iris" down in exactly the right spot in the Tigris in the mid-day bumps.

The soundings of the river we had, showed that an error of a few yards to one side or the other of a definite line would mean running aground on a sand bank.

Those who have flown large machines in the mid-day heat of Iraq will no doubt appreciate this point.

We anchored in the middle of the lake, rigged the awning over the fore part of the hull, had a meal, and left for Hinaidi about an hour before sunset.

The next day we left for Basra, spent one night there, and then left for Henjam with our passengers on board once more.

It was when within about 100 miles of Henjam that our troubles began; the starboard engine suddenly dropped from 1,900 r.p.m. to 1,600 r.p.m. The engine ran quite steadily at this speed and there was no vibration, so it was decided to carry on.

After landing at Henjam it was found that when the engine was turned the A side camshaft remained stationary. On examination, the lower camshaft drive housing was found to be fractured, thus allowing the bevel wheel at the bottom of the inclined shaft to come out of mesh.

H.M.S. "Crocus," a sloop operating in the Persian Gulf, was at Henjam, and we took the broken part on board her with a view to getting it repaired. The E.R.A.'s on board undertook the job, which was by no means an easy one. They cut away the broken parts of the upper portion of the housing and turned up a new lower portion out of solid brass. The two parts were spigotted together and pegged with four copper rivets. This work was not finished until about four o'clock the next morning, and as soon as there was sufficient light the repaired housing was refitted to the engine.

Great difficulty was again experienced in getting off at Henjam as there was only a very light wind (2-3 miles an hour) off the shore, and the tanks had been filled to their full capacity in the hopes of making a direct flight to Karachi (a distance of 630 sea miles) without refuelling at Gwadar.

The first attempt to take-off was made from a point about 1½ miles from the beach. It was found that owing to the lee of the land there was a belt of calm extending about three-quarters of a mile out from the beach, and in this calm the maximum air speed attainable was about 40 knots.

Two more attempts were made from further out to sea, but again we ran into the calm belt and had to give it up.

The final and successful attempt was made from a point about four miles from the shore.

When once in the air the load was carried without difficulty. We climbed to 4,000 feet to get away from the uncomfortable heat and all seemed well.

However, after 1¾ hours in the air, the starboard engine again dropped to 1,600 r.p.m.

By this time we were within about 20 miles of Jask and it was decided to land there rather than carry on at the risk of more serious damage to the engine.

We anchored a few hundred yards from the shore, near the Eastern Telegraph Company's Cable Station.

It was found that the copper rivets put in by the "Crocus" had sheared and that the two parts of the housing had come apart. Fortunately no further damage had been done to the engine.

The housing was taken ashore and, out of a box of scrap, some small steel bolts were found and these were screwed in in place of the copper rivets. A tap was improvised from one of these bolts to make a screw thread in the holes.

By this time it was too late to proceed and we spent the night at Jask.

We left early the next morning and reached Karachi without further incident after a flight of 6 hours 40 minutes.

SUMMARY OF FLYING TIMES AND DISTANCES—
OUTWARD TRIP.

Date.	Hours. Local time.	From.	To.	Distance. Sea miles.	Flying time. h. m.	Remarks.	Average G. S. knots.
Sept. 27	10.55	Felixstowe	Plymouth	285	3 15	—	88
,, 29	06.55	Plymouth	Hourtin	385	5 50	—	66
,, 29	14.40	Hourtin	Berre	325	4 5	—	80
,, 30	10.45	Berre	Naples	460	5 15	—	88
Oct. 1	09.30	Naples	Athens	515	5 55	—	87
,, 2	09.55	Athens	Aboukir	510	5 50	—	87
,, 8	08.00	Aboukir	Alexandretta	440	5 50	—	75
,, 11	06.40	Alexandretta	Habbaniya	445	4 55	—	90
,, 11	16.45	Habbaniya	Hinaidi	60	40	—	90
,, 12	07.00	Hinaidi	Basra	270	2 50	—	95
,, 13	06.25	Basra	Henjam	525	6 15	—	84
,, 14	09.45	Henjam	Jask	120	1 50	Forced landing	65
,, 15	06.10	Jask	Karachi	510	6 40	—	77
			Total	4850	59 10		

Average ground speed=82 knots.

The next day the improvised housing was examined and the two portions fell apart as it was taken out of the engine. It certainly would not have lasted more than another ten minutes in the air.

We had hoped to find a new engine waiting for us at Karachi, but unfortunately this was held up at Bombay and the ship carrying it was not due at Karachi for about a week.

Our programme only allowed us nine days at Karachi, so it was decided to have a complete new housing turned out of solid brass at the R.A.F. Depot in case the new engine did not arrive in time. This was done and the new housing fitted and a successful test flight made.

We were due to leave Karachi on Wednesday, October 24th, and the new engine arrived on the Sunday before and was unloaded on the Monday.

This meant that it would not be possible to change the engine without delaying the start and, as the starboard engine had run perfectly for about seven hours since the failure of the housing, and as there was no sign of any internal damage, it was decided to take a chance on the old engine. This was an unfortunate decision as we very soon found out.

The two lower camshaft drives were taken out of the new engine and one was fitted in place of the brass housing, and the other one was taken as a spare.

During our stay at Karachi our passengers had been taken a round of the R.A.F. Units in India by land aircraft.

They returned to Karachi on October 23rd and we started the return trip early the next morning.

Refuelling was carried out at Gwadar and all went well till we were about 75 miles east of Jask, flying at a height of about 2,000 feet. Suddenly oil was seen to be leaking out of the front starboard engine, and the oil pressure gradually fell away to zero. The engine was throttled right back and the other two were opened full out, but height was gradually lost at an air speed of 65 knots. Height might have been maintained at 60 knots, but at this speed there was not sufficient rudder control to keep the machine from turning to starboard.

At 500 feet the starboard engine had to be opened up to about 1,300 r.p.m. in order to keep height. This proved a bit too much for an engine running without oil, and about half an hour later, when about three miles short of Jask, the engine began to break up altogether and was switched off.

We began to lose height again and it was only by making for a gap in the palm trees that we managed to cross the narrow neck of land beyond which was the anchorage which we were trying to reach.

On examining the starboard engine a good view of a very dry and overheated looking big-end, and a broken connecting rod, was obtained through a large hole in the side of the crank case.

A message was sent to Karachi from the Cable Station asking for a land machine to take the passengers on to Iraq, and for the new engine to be sent to Jask as soon as possible.

The "Hinaidi," in which the Under-Secretary of State had done most of his tour in India, arrived the next day and landed on the aerodrome about two miles from the Cable Station.

An early start was planned for the following day, Friday, October 26th, but, owing to a little trouble with the Persian authorities, the "Hinaidi" did not get away till about two hours after dawn. However, they made a very good flight and arrived at Basra the same evening, refuelling at Bushire on the way.

A signal was received saying that our old friend the "Crocus" was arriving at Jask at dawn the next day to give us a hand and that the new engine would arrive by s.s. "Varsova" late the following Monday night.

The "Crocus" duly arrived and we began preparations for changing the engine. A very good engine lifting gear had been made by Blackburns but, although it only weighed 120 lbs., we did not like to increase the overload by carrying it in the "Iris." This gear consists of a duralumin tubular derrick about 16 feet long, made in two halves for easy stowage. At the lower end is a hemispherical socket which fits on to a ball fitting rivetted to the top of the lower front main spar in the centre section. A radius wire is taken from the top of the derrick to a pin on the top plane directly over the ball fitting, and

thus the derrick is able to swing round on the ball joint. A small hand winch is fitted to the lower half of the derrick and from this the lifting wire is led over a pulley at the top end. The length of the radius wire is such that, when the derrick is swung outwards, the pulley at the top is directly over the centre of the engine.

The problem before us was to rig up a lifting gear on these lines.

A suitable iron water pipe was found ashore and a socket was made by the "Crocus" for the lower end; also an attachment for the radius wire and for a chain purchase at the upper end.

When this gear was rigged up it looked quite a fair imitation of the real thing, moreover it worked quite well.

The "Iris" was now made fast astern of the "Crocus" and we were all given accommodation on board.

As there was practically no tide, the "Crocus" and the "Iris" both rode head to wind, but on one or two occasions, when the wind changed, the "Iris" swung before the ship did and was a little troublesome.

One evening we were all in the ward-room having a little refreshment before dinner when the boatswain came in, saluted smartly, and without a smile on his face said to me, "Your ship is coming along side, sir." We went on deck and found that the wind had changed and it was just a question of fending off the "Iris" till both "ships" had taken up the new direction of the wind.

The starboard engine was lifted off its bearers, lowered into a whaler, and hoisted on board the "Crocus" by Sunday evening.

The next day was spent in removing the two lower camshaft drives which would have to be put back into the new engine and in preparing for its installation.

The "Varsova" arrived at ten o'clock that night and the new engine was transferred to the "Crocus" by means of a native boat. This operation took about three hours. Our fitters, assisted by the E.R.A.'s, started work at once and replaced the two camshaft drives which had been removed at Karachi, and re-timed the camshafts. They worked nearly all night and by eight o'clock the next morning the new engine was lowered on to a platform made by lashing two whalers together side by side, and laying planks across them. Two hours later the engine was in position in the "Iris" and the lengthy job of connecting up all the pipes, and refitting the cowling, was begun.

By four o'clock the next day all was ready for test and we planned to get to Henjam (a distance of 120 miles) that evening.

However our troubles were not over yet and this was not to be.

A test flight revealed a water leak in the new engine, which was soon rectified and a further test flight was made. On landing we found that a valve was blowing badly in the port engine.

My crew worked right through the night, removed the cylinder with the defective valve, fitted and ground in a new valve, and all was ready by 7.30 the next morning.

During the night the E.R.A.'s dismantled the old engine and removed one cylinder in case this might be required. This cylinder we took with us as a spare.

At eight o'clock on Thursday, November 1st, we resumed our westward flight.

Henjam was reached in an hour and a half, refuelling was carried out, and two hours later we left for Basra.

On approaching Abadan (about 30 miles short of Basra) we saw a very bad dust storm ahead and had to land in the Shat-El-Arab. Two minutes after we landed, the whole place was covered with dust like a thick fog. We anchored for the night and owing to bad weather were unable to get on to Basra till the following afternoon.

About an hour after leaving Basra, the next day, the centre engine began giving trouble (obviously due to a constriction in the fuel supply) and had to be throttled down to about 1,400 r.p.m. Soon after this rain was encountered and when within about 30 miles of Hinaidi we had to fly at 500 feet to get under the rain clouds. Ten minutes later the port engine began to give out and vibrated in an alarming manner.

We had some dfficulty in getting down into the Tigris, which is very winding in this part.

On examining the petrol filters we found that they were badly choked up with fine mud. These were cleaned, but we decided to do the ten miles to Hinaidi on the water as our luck in the air did not seem to be too good.

The river was very low and we had touched bottom several times, so we sent a wireless signal to Hinaidi asking for a motor boat to be sent to pilot us along the river. In the meantime we got along as best we could. We got as far as the bend where the Diala river joins the Tigris, but we ran aground repeatedly in trying to get round this bend. Finally we took a native on board and he guided us past the shallows and then we got on quite well. Later he ran us on to a sandbank when we were doing about 15 knots and the sudden deceleration was very unpleasant. It was then that we sighted the motor boat which had been sent out to meet us.

SUMMARY OF FLYING TIMES AND DISTANCES—
HOMEWARD TRIP.

Date.	Hour. Local time.	From.	To.	Distance. Sea miles.	Flying time. h. m.	Remarks.	Average G. S. knots.
Oct. 24	06.55	Karachi	Gwadar	255	3 15	—	78
,, 24	12.30	Gwadar	Jask	253	3 20	Forced landing	76
Seven days at Jask—starboard engine changed.							
Nov. 1	08.07	Jask	Henjam	120	1 30	—	80
,, 1	11.30	Henjam	Abadan	495	5 30	—	90
,, 2	14.10	Abadan	Basra	30	20	—	90
,, 3	09.50	Basra	Zeidan	260	3 0	Forced landing. Taxied to Hinaidi	87
,, 4	11.40	Hinaidi	Habbaniya	60	50	—	72
,, 5	07.00	Habbaniya	Alexandretta	445	6 20	—	70
,, 6	06.35	Alexandretta	Aboukir	440	5 45	—	76
,, 7	07.00	Aboukir	Sollum	250	3 55	—	64
,, 7	13.00	Sollum	Bengazi	310	4 30	—	69
,, 8	07.45	Bengazi	Malta	350	5 15	—	67
,, 10	13.00	Malta	Naples	325	3 55	—	83
,, 11	06.30	Naples	Berre	460	5 45	—	80
,, 12	09.30	Berre	Hourtin	325	4 30	—	72
,, 13	07.45	Hourtin	Calshot	485	6 15	—	77
,, 14	09.25	Calshot	Felixstowe	160	2 0	—	80
			Total	5023	65 55		

Average ground speed=76 knots.

Total distance out and back, 9,873 sea miles.
Total flying time out and back, 125 hrs. 5 mins.
Average ground speed out and back=79 knots.

The difficulty of navigating this part of the river may be judged from the fact that when we first saw this motor boat, which had a draught of about 18 inches, it was itself aground on a sandbank in the middle of the river.

Hinaidi was eventually reached and we made fast to a barge which was moored across the stream near the R.A.F. Hospital.

The start next day was delayed till 11.40 a.m. by a thick morning mist.

Refuelling was carried out at lake Habbaniya to avoid the difficulty of getting off the Tigris with a big load of fuel.

Aboukir was reached on November 6th, one night being spent at Alexandretta on the way.

A rather anxious time was experienced on the flight from Habbaniya along the river to Alexandretta. After about three hours in the air the river became obscured by low clouds. A compass course was steered, and fortunately we were still over the river when, half an hour later, the clouds cleared. If the clouds had lasted longer and the river been lost, we should have been in a very awkward position as the desert here is practically featureless and we should have had to rely on dead reckoning only to hit off the mountain pass about 250 miles away.

A certain amount of anxiety was also felt as to whether the pass would be free from clouds when we reached it.

We tried to get information on this point by wireless, but were unable to do so.

Our anxiety was allayed by the presence of the Lake of Antioch, which lies a few miles to the eastward of the pass, which could be used to land on if the pass were filled up with clouds.

Our passengers were still waiting for us when we reached Aboukir and we left for Malta the next day, refuelling at Sollum and spending the night at Benghazi.

Two nights were spent at Malta, and the first whole day without any flying since leaving Jask was much appreciated.

We left Malta at 9.40 a.m., but were unable to get round or over bad rain storms, and had to return after 1½ hours in the air. We finally got away at one o'clock, flew through the Straits of Messina, and got to Naples just as it was getting dark.

We were lucky on this occasion as, with the aid of a following wind, we made good 83 knots ground speed. Without this wind we should not have been able to make Naples before dark.

The rest of the trip home was without incident, except for very bad weather south of Ushant. Low rain clouds were encountered and we had some difficulty in getting through them.

We spent one night at Naples, Berre and Hourtin, and arrived at Calshot on November 13th. Here our passengers left the "Iris," which proceeded to its home at Felixstowe the following day.

Fuel Consumption

Great difficulty was experienced in estimating with any degree of accuracy the quantities of fuel used on the various flights. The exact contents of the barrels in which the fuel was usually supplied, was often doubtful, and there was always a certain amount left in the barrels which could not be sucked out by the pump.

The average fuel consumption for the whole trip appeared to be about 95 gallons an hour.

Refuelling

The refuelling was carried out by means of a Zwicky pump, 4in. bore by 4in. stroke, through 1in. internal diameter petroflex hose. The lengths of hose carried were two 15 feet lengths and one 30 feet length. A fine gauze filter was fitted on the suction side of the pump.

The pump was mounted on a special mounting on the starboard lower centre section, and the delivery from the pump was through one 15 feet length of hose to the junction box in the middle of the lower centre section.

The fuel was usually supplied in 50-gallon barrels, and this was found to be by far the most convenient form.

At Hinaidi, and at all refuelling places east of Hinaidi, the fuel was supplied in 5-gallon drums.

A lot of time was always wasted in opening these drums, as there seemed to be no other way of unscrewing the stoppers except by means of a cold chisel and a hammer. The opening in the drums was too small to insert the hose, and the fuel had to be poured from the drums into some form of container (a small galvanised iron bath was used at Basra), and the fuel sucked from this by the pump.

Another disadvantage of the 5-gallon drums is that the fuel did not seem to be so clean as that in the 50-gallon barrels.

Also there was always a certain amount of dust and dirt on the top of the drums, and as the fuel was poured out some of this was carried into the container with the fuel. This was specially noticeable at Basra on the return trip.

The fuel was brought alongside the flying boat in some form of boat or motor boat.

At Berre the fuel was brought alongside on a large flat-topped raft and this was found to be a very convenient arrangement.

The position of the pump mounting was very convenient and was such that two men could operate the pump if necessary.

One man could operate the pump fairly easily, but usually there was plenty of labour available, and it was operated by two men.

Remarks on the "Iris"

I have very little but praise for the "Iris" and I don't know of any other flying boat in which I should have preferred to carry out a trip of this nature.

The "Iris" is good on all controls and is not at all tiring to fly, in spite of her size.

We had very little bumpy weather to contend with, but on one occasion a strong wind, blowing from the high mountains of Corsica, caused some very violent disturbances in the air, and even then the "Iris" proved remarkably controllable, although the motion was anything but pleasant.

The pilots' cockpit is very roomy and comfortable and access to the hull from both pilot's seats is easy. There is even room for a third person to sit between the two pilots, and just a little behind them, and from this position a very good view can be obtained.

The internal arrangements of the hull are good and include a navigator's desk large enough to take an ordinary Admiralty chart and ample accommodation for the wireless operator and his apparatus.

Two wicker armchairs were provided for the passengers and a small folding table which was used for meals in the air.

Four bunks were rigged amidships of which two were usually full of light kit, overcoats, etc.

The other two were frequently used for the purpose for which they were designed and appeared to be very comfortable.

Further aft there was a "Clyde" cooker, fitted with two Primus stoves. This came in for quite a lot of use, both in the air and on the water.

When flying, the noise inside the hull was not excessive, and it was possible to carry on a conversation without difficulty. Conversation was also possible in the pilots' cockpit, but was not so easy.

The ability to move about the boat with ease, and have an occasional rest down below, as it were, and to have really comfortable meals, was found to reduce fatigue to a minimum.

The difficulty of taking-off with no wind must now be considered. Designers of flying boat hulls have always been faced with the difficulty of hitting off the happy medium between the flat bottom for quick take-off and the deep V bottom for easing the shock of landing.

The maximum load at which the "Iris" will take-off a calm sea with no wind appears to be about 28,000 lbs.

The maximum load at which she has ever taken off is 33,000 lbs. with a wind of about 15 miles an hour, and it has been estimated from test figures that she would be able to fly at a load of about 40,000 lbs.

Hence the difference between the maximum flying load and the maximum load at which it is possible to get off with no wind is about 12,000 lbs.

The bottom of the "Iris" is of a deep V section and the normal draught is 3ft. 10in. The sides of the V are so concave that, where the plates join the keel, they are nearly vertical.

On landing, this sharp keel cuts into the water and reduces landing shock to a minimum, but as soon as the keel touches the water, the deceleration is very noticeable, which shows that the resistance in the water is considerable.

It is considered that this water resistance accounts to a large extent for the difficulty of taking-off with no wind.

It must also be remembered that when the boat is nearly off the water, there is practically no lift, due to hydroplaning as the submersed portion of the hull forms such an acute angle.

A few take-off figures may be of interest.

Load, lbs.	Wind speed miles/hour.	State of sea.	Time of take-off. Secs.
27,000	0	Calm	31
28,000	0	,,	60
29,000	3	,,	38
30,000	4	,,	44
32,000	4	,,	54
33,000	15	,,	62

It will be seen that a wind of only three miles an hour reduces the time of take-off with no wind by 22 seconds even when the load has been increased by 1,000 lbs. This seems to suggest that only a slight modification to the hull shape might result in a big improvement in the take-off.

This idea is confirmed in practice.

On the occasions when we failed to get off, an air speed of 40 knots could be attained without difficulty, but this speed could not be exceeded.

The impression felt was that if the keel would skim over the water, instead of cutting through it, and give a little lift instead of only drag, the speed would increase and reach take-off speed.

Another disadvantage of the deep V is that if the "Iris" is landed with even a slight drift, an uncontrollable turn of about 120° takes place a few seconds after touching the water; the danger of this when landing in a river, where it may be necessary to land slightly out of wind, is obvious.

This is probably due to the fact that, as soon as the keel touches it bites into the water, and a side pressure is set up due to the drift, and as the hull sinks deeper into the water, the centre of pressure moves further forward. As soon as this centre of pressure is forward of the c.g. of the machine, a horizontal turning moment is set up.

It is thought that this difficulty might be overcome by rounding off the keel a little so that it could skim sideways over the water and thus take up the drift more gradually.

A great advantage of the deep V was experienced when taking-off in a swell with a fairly strong wind. Under these conditions a flatter bottomed boat would probably have been bounced into the air before flying speed had been attained owing to the bows of the boat being lifted on striking a swell.

With the "Iris" this was not the case. The bows cut through the swell and any tendency of the bows to be lifted by the swell could easily be counteracted by the elevator.

This feature is considered to be of very great advantage.

It is suggested that the take-off of the "Iris" might be improved by making the V a little flatter and by rounding off the keel in the neighbourhood of the step, but that the sharp V should be retained for the forward part of the hull in order to retain the good qualities when taking-off in a swell.

When flying with one engine throttled right back, it was found necessary to hold the rudder hard over to keep straight, and even then, at an air speed of less than 65 knots, the rudder control was not sufficient to prevent turning towards the throttled engine.

It is considered that in the design of twin and multi-engined aircraft, some provision should be made for flying with one engine stopped.

An adjustable tail fin is suggested, or an extra rudder, operated independently of the rudder bar. Without some such provision the only chance of flying with one engine stopped is when the air is quite calm, as with the rudder hard over, there is no rudder control left in one direction to counteract bumps.

An adjustable tail fin would also be very useful for normal flying as nearly all multi-engined aircraft have a tendency to turn in one direction or the other. This involves either a constant pressure on the rudder bar which, though slight, may be very tiring on a long flight, or throttling down one engine.

It was found on the "Iris" that the port engine had to be run at about 50 r.p.m. less than the starboard in order to keep straight without any rudder.

The fore and aft trimming gear of the "Iris" is considered to be very good. From the design point of view it seems to be much simpler than the adjustable tail plane, and it is very effective. The loss of trim caused by one man moving from the pilot's cockpit to the cockpit behind the rudders was counteracted by moving the elevator lever through about one third of its range.

Engines

Apart from the troubles mentioned, the "Condor" engines ran very well indeed and gave no anxiety.

The defective starboard engine was returned to Karachi and details of the internal failure are not available, but it seems almost certain that the failure was due to the strain of running for about two hours with six cylinders cut out.

The cause of the fracture of the camshaft drive housing was not apparent, but there have been other failures of the same part and I understand that the design has now been modified.

Navigation

For the greater part of the cruise land was in sight and the air pilotage presented no difficulties.

The only long sea crossings without sighting land were from Cape Sidero (Crete) to Aboukir, 300 sea miles; from Aboukir to Farmagusta (Cyprus), 300 sea miles; and from Bengazi to Malta, 350 sea miles.

Cape Sidero lies on the direct line from Athens to Aboukir, and the drift was checked when approaching this point.

In order to check drift when approaching a point, a vertical wire was fitted near the middle of the pilots' windscreen and a vertical line was marked on the inside of the front cockpit so that a line joining this mark and the vertical wire was parallel to the fore and aft axis of the aircraft.

Other marks were made every 5° up to 20° to port and starboard.

The drift was found by trial and error. For instance, suppose the drift was thought to be 10° to starboard, then the aircraft was turned till the point being approached coincided with the starboard 10° mark in the front cockpit when sighted from the vertical wire. The compass was set in this position and the same course steered for a few minutes.

If, after this time, the point of land was still on the 10° mark, it was clear that the drift had been correctly estimated. If the angle had increased, then it showed that too much was being allowed for drift, and vice-versa.

When the drift had been determined in this way, and the appropriate correction had been made on the compass, the same compass course was maintained after having passed directly over the point of land. Back bearings were then taken on the point by means of the 0.3 compass, and the compass course altered again if necessary, until a constant back bearing of the required magnetic truck + or −180° was observed.

When out of sight of land the drift was checked at intervals by means of cartons containing aluminium dust. One of these was thrown overboard and a back bearing taken on the mark made by the aluminium dust on the surface of the water.

This method was found to be quite satisfactory, except when the surface of the sea was covered with white crests, when it was difficult to distinguish the mark made by the aluminium dust from the white crests. The appearance of the two from the air is very similar.

On one occasion a carton was dropped from 4,000 feet, but owing to its lightness and consequent slow rate of fall, it was too far behind when it reached the water.

It was found in practice that about 1,500 feet was the maximum height for good results.

Theoretically, to get accurate results by this method, the carton should strike the water directly underneath the aircraft, hence, the greater the height the less accurate is the result.

During this process the aircraft must be kept on a very steady course. Hence, when the air is bumpy, the results are not so reliable.

Back bearings were taken from one of the two side ports in the hull just aft of the trailing edge. The compass standard was screwed to an angle bracket which fitted the standard drift plate fitting. These fittings were rivetted to the outside of the hull, just below the middle of the side ports. This position was found to be very convenient.

During the flight from Bengazi to Malta, D.F. bearings were sent from Malta every half an hour. The first three D.F. bearings were second class and indicated that the aircraft was slightly to the eastward of the correct track. The fourth bearing, received 2 hours 15 minutes after leaving Bengazi, was first class, and showed that the aircraft was exactly on the line joining Bengazi and

Malta. One hour after this the bearing had altered one degree. Shortly after this the wind (which was on the port beam) increased in strength, and a back bearing on the aluminium dust showed that the drift had increased from 11° to 16°, and the compass course was altered accordingly.

Four and a half hours after leaving (*i.e.*, 45 minutes before landing at Malta) a D.F. bearing showed that the aircraft had drifted to leeward of the correct track and the compass course was altered 10°. Half an hour later Malta was sighted.

Conclusion

In conclusion, I should like to express my gratitude for the kindness and help we received wherever we went.

All possible assistance was given at the foreign seaplane stations we visited, and I should like specially to mention the Italians who went to a lot of trouble in sending us particulars and photographs showing exactly where we had to land at Naples. When we left Naples the Italians put up a very good escort consisting of three single-engined flying boats.

Our greatest thanks are due to the Navy, as represented by H.M.S. "Crocus," for the invaluable assistance they gave us at Henjam and Jask.

Finally, I should like to express my regret for the inconveniences our passengers were sometimes called upon to put up with, and to thank them for the sporting spirit which they showed when things happened not quite according to programme.

Discussion

The President: Not only was this the first time that Squadron Leader Scott had given a paper before the Society but it was the first lecture he had ever given. The author had apologised for the lack of technical information in the paper, but there was no need for such an apology. What one wanted to learn from these extended cruises was information of a practical character which people like Major Rennie—the designer of the "Iris"—and others could benefit from, so that improved equipment might follow as a result of the experience that had been gained. Captain Cave-Browne-Cave was present, and he would ask him to open the discussion.

Captain H. M. Cave-Browne-Cave: The author had referred, in his opening remarks, to the cruise of the Far East Flight, and had said that the cruise of the "Iris" to India and back was much smaller. It should be remembered that this cruise to India and back was made with types of aircraft and engines which had been used comparatively little in the R.A.F., and consequently all the defects had not yet been found and put right; the programme to which this cruise was made allowed little time for the maintenance of the aircraft and engines, and passengers and baggage were carried in addition to the crew. In fact, it was found that, with the normal load carried, this three-engined boat could only "take-off" if there was a wind, and could not maintain horizontal flight if one engine was stopped or throttled right back. This must have added very greatly to the anxiety and difficulty of the cruise. Other things being equal the chance of engine failure was proportional to the number of engines in use, and the three-engined boat was at a disadvantage in this respect, compared to a twin or single-engined boat, unless it could fly well on any two of its three engines. The author had stated in the paper that this boat could stay in the air at a total weight of about 40,000 lbs. He thought it would be unwise to use this boat at such a weight, as the complications and head resistance of the three-

engine arrangement was only justified if the boat could keep flying with one engine stopped. He agreed with what the author said about the steep V hull bottom. He thought the great advantage of the steep V was that the pilot could keep the boat on the water till flying speed was reached when " taking-off " into a swell, whereas a comparatively flat bottom boat might be thrown into the air stalled, with the chance of one wing dropping and a serious accident. The use of the automatic slot for lateral control should reduce this risk. The author had pointed out the disadvantages of the very steep V used in " Iris." He thought the strong tendency of the boat to swing when landed side to wind was a serious disadvantage, and might often prevent the boat operating from narrow channels, rivers and small harbours. In the open sea also he thought the ability to " take-off " or " land " along the sea, even if side to wind, would enable a flying boat to operate under much worse conditions than would be possible if it was forced always to " take-off " or " land " head to wind. With reference to the author's experiences with the Persian authorities, the Far East Flight had considerable difficulty in getting permission to visit the Persian parts, and on arrival at Bushire they had all been put in quarantine and forbidden to land; this saved paying official calls. Everyone would sympathise with the author and his crew finding a leaking valve in the spare engine after all the work of getting and fitting the engine; their having to work all night to rectify this defect draws attention to the large amount of work caused by even a minor valve defect in aircraft with overhead camshafts. The later water-cooled aero engines have overhead valves and camshafts and monoblock cylinder construction. It is very important that the valves of these engines should not require renewal or grinding in between engine overhauls, as it is difficult and risky to remove and replace a heavy cylinder block in a flying boat rolling at its moorings. As regards refuelling, the Far East Flight also found that no proper openers were supplied with the 5-gallon drums, and that the fuel in them was more likely to be dirty than was the case with other containers; they avoided the difficulty of having to empty the drums into a container before pumping into the aircraft tank by carrying a copper tube which could be passed in through the opening in the end of the drum and screwed to the pump suction hose. This also avoided the risk of dirt on the drum getting into the petrol; this copper tube was fitted with a small filter, which could be quickly removed for cleaning. The author had used the same system for navigation as was used by the Far East Flight, and that Flight also found that it was only suitable up to about 1,500 ft. Further, when flying away from the sun it was found very difficult to pick out the patch of aluminium dust on the glittering surface of the sea.

Wing Commander HYNES: He congratulated the author on the ingenuity and resource shown in replacing the damaged engine without any real facilities for doing it. It was difficult enough to do this sort of thing on land without proper tackle, but it must have been infinitely more difficult to handle an engine of the size of the Condor III. under the extraordinarily difficult conditions that existed in this case. The other point he had intended speaking on had already been mentioned by Group Captain Cave-Browne-Cave, and that was that it seemed entirely wrong to have a three-engine boat that would not fly and fly reasonably well on two engines. As had been suggested, this simply meant an increase of the risk of failure by something of the order of three times as compared with a single engine. From what the author had said we must draw also the inference that the three engines in this machine had had to be worked well up to a high proportion of their full load, and that being so the risk of breakdown was increased.

Major J. D. RENNIE: It will be noticed that the average ground speed is about five times that of a passenger ship operating on the India route. To take full advantage of this in a commercial flying boat service would mean flying by day and night. In the present stage of aircraft development, this advantage in

speed is in no small measure counterbalanced by the accompanying noise and general discomfort. The steamship, of all modern means of transport, still remains the most peaceful and restful. The reduction of, especially noise, to even an amount comparable with an express train, would provide the greatest stimulus to the success and popularity of long distance air transport.

He would like to make a few comments on the lecturer's remarks on "Iris." In the first place, it should be remembered that, while flying boats, the F type boats in particular, were developed and fairly extensively used in this country during the War, all further development practically stopped on the termination of the War, and it was not until early in 1924 there were signs of further official interest in this type of aircraft. "Iris" was one of the first large boats to be ordered by the Air Ministry after this period of stagnation. The design is therefore five years old, and in view of the progress which has taken place within the last five years, she cannot be looked upon as representing the latest in flying boats. In the three "Irises" at present under construction, advantage has been taken of the experience gained with "Iris II." in the Service, and as far as possible the design has been modified accordingly.

The lecturer spoke of the difficulty he experienced in taking-off in a calm sea and in wind, and attributes this to the water drag of the deep Vee sections of the hull bottom. Obviously, so long as the keel is in the water there will be a resistance. Tank tests have been carried out on the "Iris" model by the Air Ministry to obtain information on this matter. The model was towed at two attitudes, and with varying amounts of immersion of the keel. While the data obtained is not available, it may be said definitely that the drag of the keel does not supply the whole solution. The real trouble lies in the fact, which was known in the layout stage of the design, that it was not possible to use the best propeller diameter for take-off and climb. If, instead of the present 12ft. 6in. diameter four-blader propellers, the design, as a whole, admitted of 15ft. 6in. two-blader propellers, the take-off would be beyond criticism. In the same connection, it should be stated that in the design of "Iris," especially with regard to the hull, an endeavour was made to produce a boat possessing seaworthy qualities superior to what was then generally accepted as possible. Actual experience in service has shown that this very important feature has been obtained. Quoting from the official report, "Its excellent seaworthy qualities constituting a noteworthy feature." However, as the lecturer says, it may be possible to obtain a better compromise to meet the calm sea and no wind conditions.

There is another point which may bear on the difficulty of taking-off. It is possible that the atmospheric conditions prevailing at the time, at sea level, may have been equivalent to several thousands of feet in the standard atmosphere. In which case there would be a considerable reduction in available horse-power for taking-off.

The engine trouble experienced was most unfortunate, but if it had not occurred it might not have become known, that under tropical conditions, if a wing engine cut out, the rudder control was inadequate. In this country tests had been carried out with a wing engine throttled right down, and the control was reported to be quite satisfactory

With regard to the engines. In fairness to Messrs. Rolls-Royce, the makers, he thought he was right in saying that the design is obsolete, and has not been in production for several years. It is to their credit that, apart from this mishap, the "Condor" engine had given excellent service in "Iris."

Captain TYMMS: If there was one thing more than another which interested him in this paper it was the incident of the Persian Corporal of Police at Jask.

This incident was important as an indication of the conditions existing, not only in Persia, but in many countries nearer home. If obstructions of this nature are experienced by a Cabinet Minister and a Foreign Secretary flying in a Royal Air Force machine, it must be quite clear that a civil aviator must experience considerably greater difficulties—difficulties not of a practical but of a political nature. In a large part of Europe similar political difficulties to the freedom of flight will be found. There are prohibited areas practically encircling whole countries. There is one interesting example. There are two adjoining countries both of whom have declared prohibited areas along the whole common frontier. In order that one might fly out of the country each had declared aerial corridors through their own prohibited areas; the only trouble was that the corridors did not meet.

He called attention to this fact because of the desirability of throwing the light of publicity on these conditions, with the hope that such publicity will ultimately result in a modification of the conditions.

There was one other point which interested him, and that was with regard to the navigation. The author had said that he was not able to use aluminium powder on the sea when there were white capped waves; he personally wondered why the author wanted to do so. He assumed that the design of the "Iris" precluded the use of an ordinary drift bearing plate, because if such were carried it would be quite possible to obtain the drift by a vertical sight on the foam itself.

The PRESIDENT: He suggested that it was bad policy to send a machine on a cruise of such importance fitted with engines that had always given a good deal of trouble. It gave a wrong and totally undeserved impression. However, that had nothing to do with the author or his excellent paper. A great deal had been said about the particular conditions met with along our principal route to the East. The Society had been presented with information from time to time with regard to land machines operating under these conditions, but until they had had Captain Cave-Browne-Cave's paper, and now the one by Squadron Leader Scott, they had not fully appreciated the conditions that had to be met with in operating flying boats. He wished, with others, to congratulate the author and his crew on the ingenuity they showed in changing engines at sea. There was the point raised by Captain Tymms of the Persian corporal who seemed to have made things very difficult for the Under-Secretary of State and an Air Vice-Marshal and the others with the author, and it seemed to point to the greater ease of operating flying boats as compared with land machines. Captain Cave-Browne-Cave, when passing through the Persian Gulf, was put into quarantine and was told that he must not go ashore, so he just stayed on board and left when he was ready, whereas if he had been in an aeroplane he might have been detained for some days!

REPLY TO DISCUSSION

Squadron Leader SCOTT: With regard to the load of 40,000 lbs., mentioned by Captain Cave-Browne-Cave, he admitted that that probably would be a little too much. That load was got at by calculation from figures which he had obtained from Felixstowe, and on those figures the "Iris" should be able to climb at the rate of 100ft. per minute, but he was a little doubtful about it. As to not seeing the aluminium dust when flying away from the sun, he could quite understand that, but it so happened that these conditions never arose during this particular flight. The sun was always in a convenient position. Major Rennie, in his written communication, stated that if 15ft. propellers were used the "take-off" of the "Iris" would be all right. No doubt that was quite true, and with the deep V would give comfortable landings. He agreed with Wing Commander Hynes that a three-engine machine should be able to fly with two engines. It

should be remembered that during the flight it was very hot, and out there the conditions were very different from what they were here. Nevertheless, if the machine was to do its job it should be able to fly with two engines under the conditions in which it had to operate. Captain Tymms had suggested the use of the drift plate, but he was sorry to say that these were not carried, otherwise they would have been satisfactory for getting the drift, although the fact that the white crests were also moving would give a slight error, but probably nothing to worry about.

TESTS ON MODEL PROPELLERS

BY

JOHN L. HODGSON, B.Sc., Assoc.M.Inst.C.E.

In the July Journal a number of figures from the reprint of the above paper read before the Institution of Automobile Engineers in March, 1917, were unfortunately omitted. The figures are printed below. In the reprint of the paper by Signor V. Isacco (Reprint No. 43) these figures have been placed in their proper position in the text.

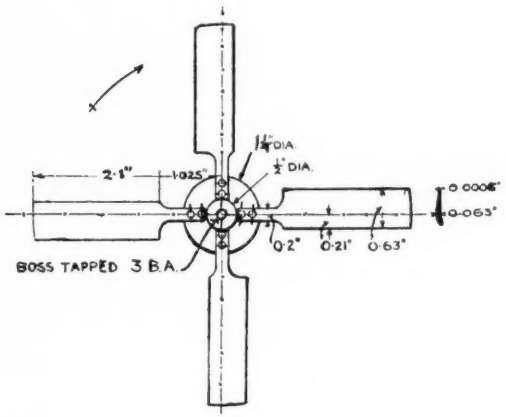


Fig. 1.

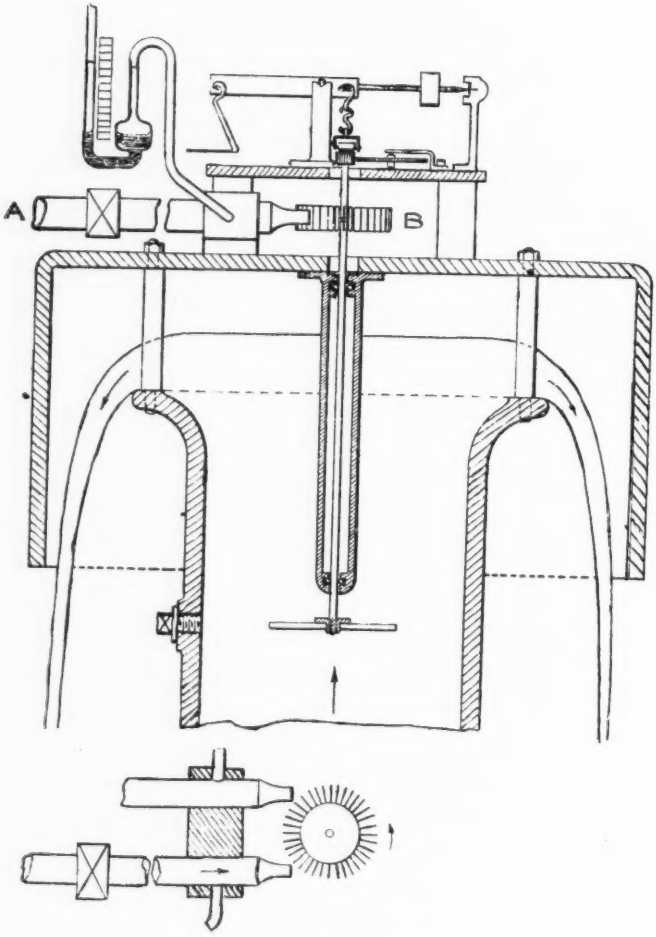


FIG. 2.

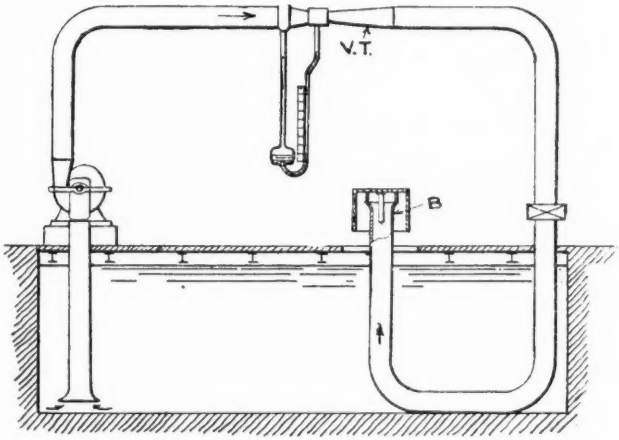


FIG. 3.

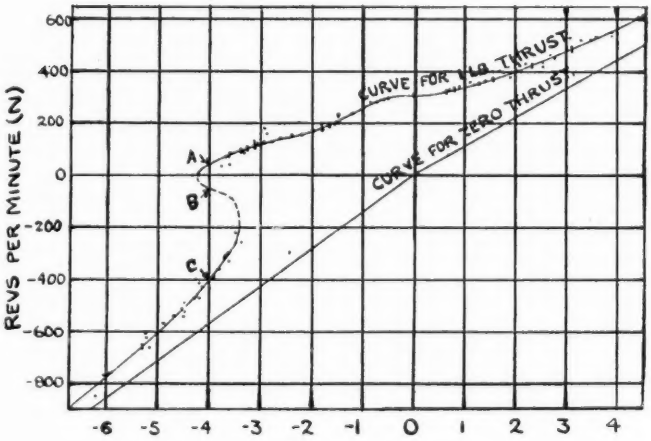


FIG. 4.

Velocity of advance (V) in water in feet per second.

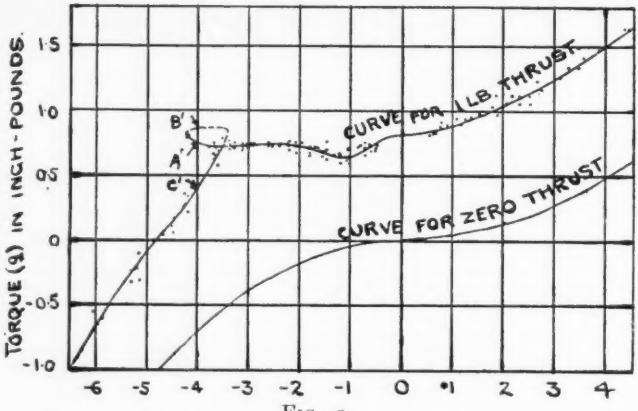
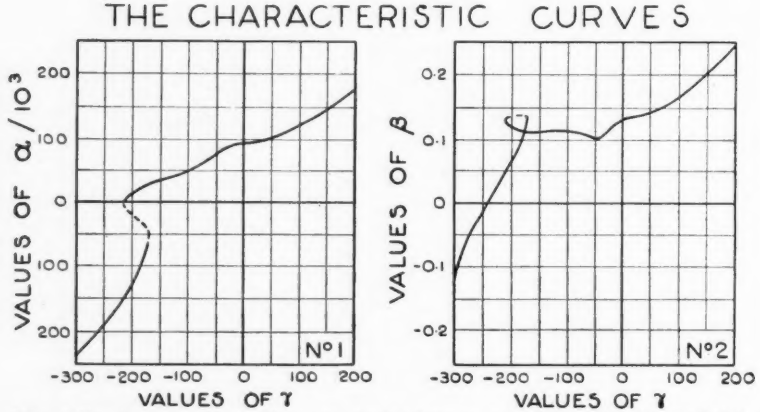


FIG. 5.

Velocity of advance (V) in water in feet per second.



The following curves, which are derived from equations (1), (2) and (3) and the "characteristic curves" (Nos. 1 and 2 above), enable any two of the six variables T, N, q, V, W and D to be determined when the remaining four are known.

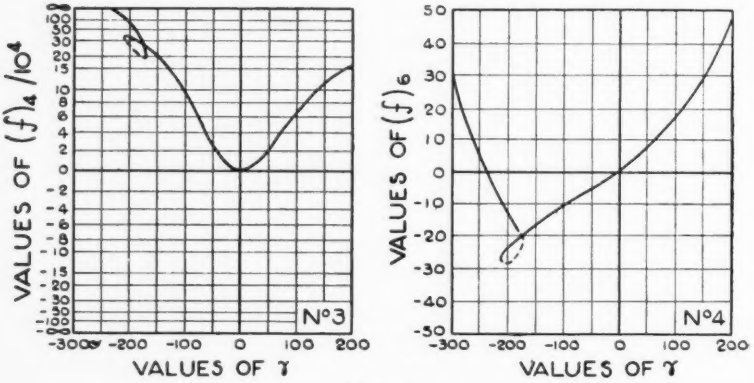


FIG. 6.

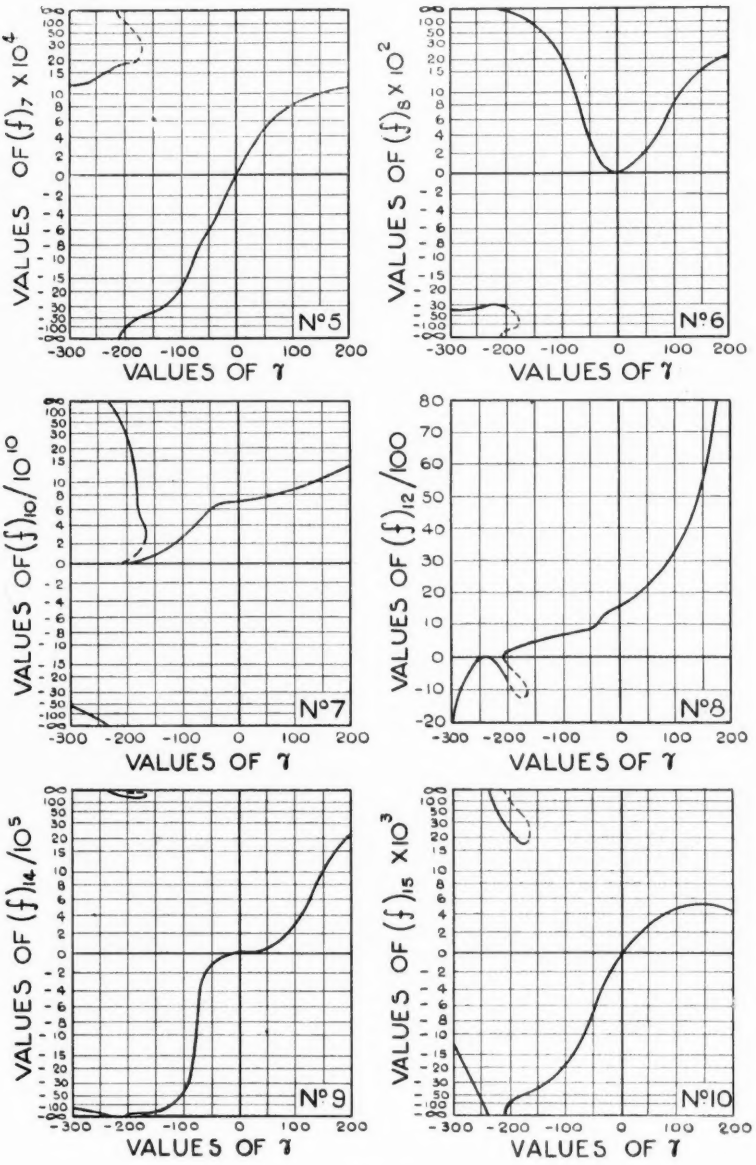

VALUES OF $(f)_7 \times 10^4$
VALUES OF γ
N°5
VALUES OF $(f)_8 \times 10^2$
VALUES OF γ
N°6
VALUES OF $(f)_{10}/10^{10}$
VALUES OF γ
N°7
VALUES OF $(f)_{12}/100$
VALUES OF γ
N°8
VALUES OF $(f)_{14}/10^5$
VALUES OF γ
N°9
VALUES OF $(f)_{15} \times 10^3$
VALUES OF γ
N°10

FIG. 7.

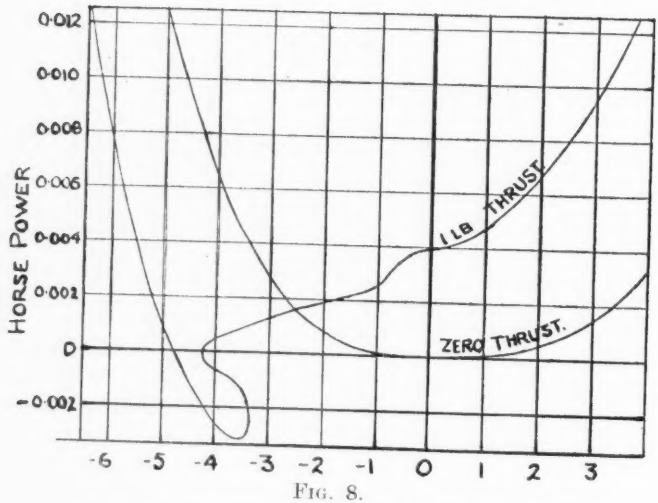


FIG. 8.

Velocity of advance (V) in water in feet per second.

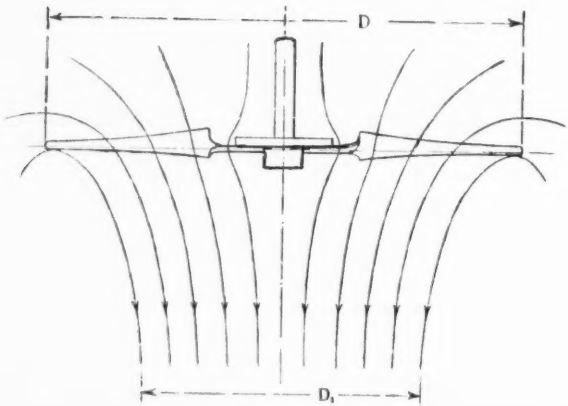


FIG. 9.

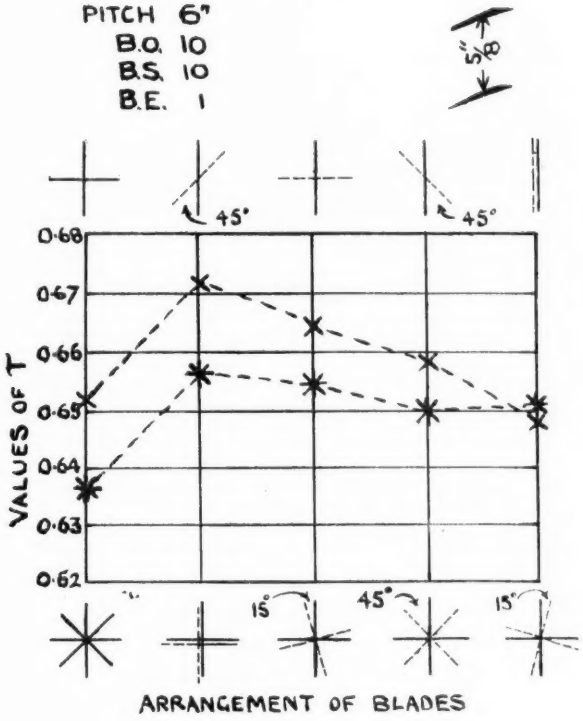

PITCH 6"
B.O. 10
B.S. 10
B.E. 1
5/8"
45°
45°
0·68
0·67
0·66
0·65
0·64
0·63
0·62
VALUES OF T
15°
45°
15°
ARRANGEMENT OF BLADES

FIG. 15.

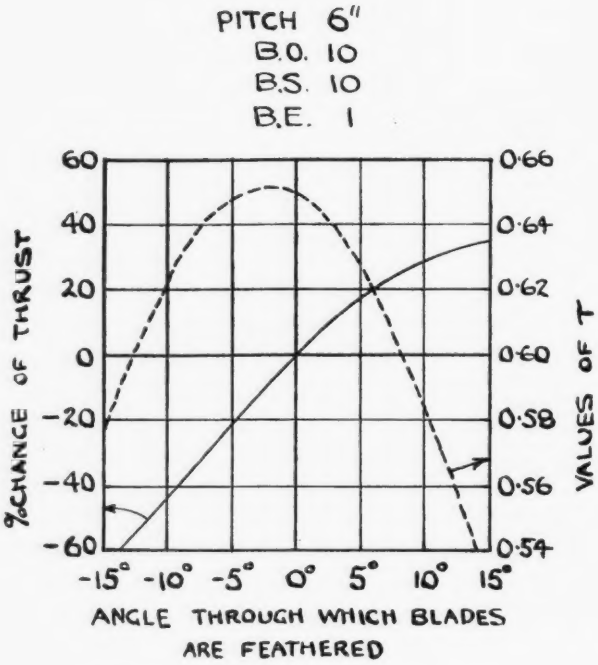

PITCH 6"
B.O. 10
B.S. 10
B.E. 1
60
40
20
0
-20
-40
-60
% CHANGE OF THRUST
0·66
0·64
0·62
0·60
0·58
0·56
0·54
VALUES OF T
-15° -10° -5° 0° 5° 10° 15°
ANGLE THROUGH WHICH BLADES ARE FEATHERED

FIG. 16.